Krichever: Isomonodromy equations on algebraic curves and Whitham equations

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Integrable Systems.... KdV eqn's, waves in shallow water:

$$u_t = u u_x + u_{xxx}$$

Non-linear Schrödinger (NLS)

$$i\psi_t = \psi_{xx} + |\psi|^2 \psi$$

Sine-Gordon eq'n

$$u_{tt} - u_{xx} = \sin u$$

Toda Lattice

$$\ddot{x}_i = e^{x_i - x_{i+1}} - e^{x_{i-1} - x_i}$$

KP

(waves in plasma)

$$\sigma^2 u_{yy} = (u_t - u u_x - u_{xxx})_x$$

What is in common?

{integrable systems} $\Longleftrightarrow$ {Can be presented as a compatibility condition of an over-determined system of linear eq'ns}

Example $L\psi = E\psi$ (linear op, energy)

$(\partial_t - A)\psi = 0$

} an over determined system

compatibility: $[L, \partial_t - A] = 0$

$\Longleftrightarrow L' = [A, L]$ ⟸ "Lax Form"

$$L = (\partial_x^2 + u(x,t)) \qquad A = (\partial_x^3 + \tfrac{3}{2} u \partial_x + \tfrac{3}{4} u_x)$$

Compatibility becomes KdV, up to minor normalizations.

Aside Zacharov-Shabat ... a similar construction for NLS

Why is it good?

Get conserved quantities:

$$(\operatorname{tr} L^k)' = 0$$