elucidated Alek. Tor. ...

Dror Bar-Natan: Talks: Bern-131104:

ω :=http://www.math.toronto.edu/~drorbn/Talks/Bern-131104

The Kashiwara-Vergne Problem and Topology

Abstract. I will describe a general machine, a close cousin of Taylor's theorem, whose inputs are topics in topology and whose outputs are problems in algebra. There are many inputs the machine can take, and many outputs it produces, but I will concentrate on just one input/output pair. When fed with a certain class of knotted 2-dimensional objects in 4-dimensional space, it outputs the Kashiwara-Vergne Problem (1978, solved ~~by~~ Alekseev-Meinrenken, 2006), a problem about convolutions on Lie groups and Lie algebras.

add v.f.s

add pictures

The Kashiwara-Vergne Conjecture ~~(1978, proven Alekseev-Meinrenken, 2006)~~. There exist two series F and G in the completed free Lie algebra in generators x and y so that

$$x + y - \log e^y e^x = (1 - e^{-\operatorname{ad} x})F + (e^{\operatorname{ad} y} - 1)G$$

and so that with $z = \log e^x e^y$,

$$\operatorname{tr}(\operatorname{ad} x)\partial_x F + \operatorname{tr}(\operatorname{ad} y)\partial_y G = \frac{1}{2}\operatorname{tr}\left(\frac{\operatorname{ad} x}{e^{\operatorname{ad} x} - 1} + \frac{\operatorname{ad} y}{e^{\operatorname{ad} y} - 1} - \frac{\operatorname{ad} z}{e^{\operatorname{ad} z} - 1} - 1\right).$$

Implies the loosely-stated convolutions statement: Convolutions of invariant functions on a Lie group agree with convolutions of invariant functions on its Lie algebra. More accurately, let G be a finite dimensional Lie group and let $\mathfrak{g}$ be its Lie algebra, let $j : \mathfrak{g} \to \mathbb{R}$ be the Jacobian of the exponential map $\exp : \mathfrak{g} \to G$, and let $\Phi : \operatorname{Fun}(G) \to \operatorname{Fun}(\mathfrak{g})$ be given by $\Phi(f)(x) := j^{1/2}(x) f(\exp x)$. Then if $f, g \in \operatorname{Fun}(G)$ are Ad-invariant and supported near the identity, then

$$\Phi(f) \star \Phi(g) = \Phi(f \star g).$$

add "Machine"

+ box with pixelated images

use a Bern bear

+ Taylor.

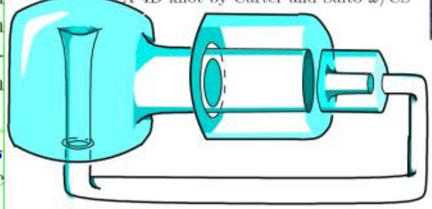
Add Theorem

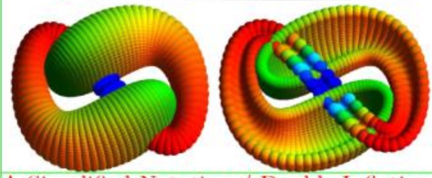
Dancso

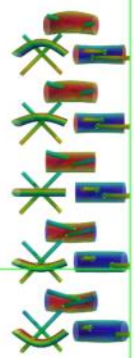
Ester Dalvit
ω/Dal

4D Knots.

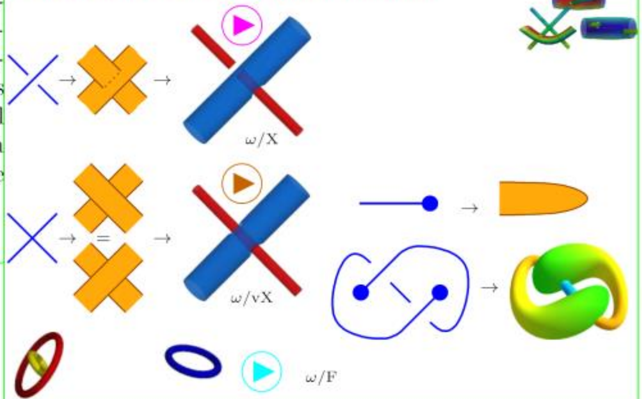
A 4D knot by Carter and Saito ω/CS



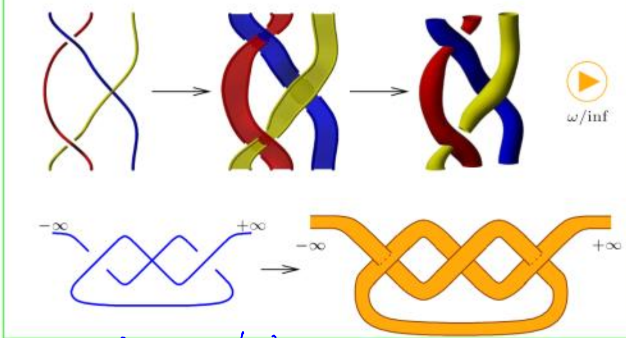




A Simplified Notation / Double Inflation



The Double Inflation Procedure δ.



Add vertices
Add PA, all relations, ops.

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org