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Braids and Associators, problem set 7 — by Dror Bar-Natan

Online: `http://drorbn.net/AcademicPensieve/2013-11/MZV_ex7.pdf`.

1. Show that the Kontsevich/KZ integral $Z\colon PB_n \to \operatorname{gr} PB_n$ satisfies property U: for $D \in \mathcal{G}_m\mathcal{A}_n$, one has $(\operatorname{gr} Z)(\pi(D)) = D$. You may take for granted that it is well defined — namely that it is invariant under deformations of geometrical braids.

2. Show that the Kontsevich/KZ integral is multiplicative.

3. Verify Arnold's identity: if $\omega_{ij} = \dfrac{dz_i - dz_j}{z_i - z_j}$, then $\omega_{12} \wedge \omega_{23} + \omega_{23} \wedge \omega_{31} + \omega_{31} \wedge \omega_{12} = 0$. (This will be used within the proof of the invariance of Z).

4. Recall from the last assignment (exercise 3) that there is a naturally-defined "co-product" map $\Delta_{A_n}\colon A_n \to A_n \otimes A_n$, where $A_n = \operatorname{gr} \mathbb{Q}PB_n$.

(i) Describe Δ_{A_n} explicitly; in particular, demonstrate that you know how to compute Δ_{A_n} by computing $\Delta_{A_n}(t^{12}t^{13}t^{23})$.

(ii) Show that the Kontsevich/KZ integral is co-homomorphic in the sense of the last assignment.