



Random Wisdom.

- In double trees, the punctured side is always above the capped side.

General Equations.

All verifications at DoubleTreeVerification.nb

- ✓1. R4  $R_{23}R_{13}V = VR_{12,3}$
- ✓2. Twist  $V\Theta_{12} = RV^{21}$
- ✓3. NonDeg  $V//d\eta^1 = V//d\eta^2 = 1$
- ✓4. Cup  $C = C//dS$
- ✓5.  $\Phi := V_{12,3}^{-1}V^{-1}V_{23}V_{1,23}$
- ✓6.  $\square$   $\Phi\Phi_{1,23,4}\Phi_{234} = \Phi_{12,3,4}\Phi_{1,2,3,4}$
- 7.  $\square_{\pm}$   $\Theta_{12,3}^{\pm} = \Phi\Theta_{23}^{\pm}\Phi_{132}^{-1}\Theta_{13}^{\pm}\Phi_{312}$
- ✓8. HF4 $\Phi$   $\Phi\Phi_{321} = 1$
- ✓9. VF4 $\Phi$   $\Phi(\Phi//dS^{123}) = 1$
- ✓10. OH  $\Phi_{01,2,3}//dS^{23}//dm_0^{03}//dm_1^{12} = 1$
- ✓11.  $\nu := (\Phi//dS^2//dm_1^{321})^{-1}$
- ✓12.  $C^4 = \nu$
- ✓13.  $V//h\eta^1//t\eta^2//dm_1^{21} = R_{11}^{1/2}$
- ✓14.  $V//h\eta^2//t\eta^1//dm_1^{12} = 1$
- ✓15.  $V//t\eta^2//dS^1//dm_1^{21} = R_{11}^{-1/2}$
- ✓16.  $V//t\eta^1//dS^2//hm_1^{12} = 1$
- ✓17. The Buckle  $Z_{\otimes} := \Phi_{13,2,4}^{-1}\Phi_{132}\Theta_{32}\Phi^{-1}\Phi_{12,3,4}$
- ✓18. Buckle2V  $V_{\otimes} := Z_{\otimes}//t\eta^1h\eta^2//dm_1^{12}//t\eta^3h\eta^4//dm_2^{34}$

To Do. • The double tree procedure  $\varphi$ . • Well-definedness. •  $\varphi = \varphi\varphi$ ? • Behaviour under disjoint unions. •  $\varphi(\uparrow)$ . • Behaviour under strand concatenation. • Behaviour under annulus unzip. •  $\varphi(\uparrow)$ . •  $\varphi(\uparrow\uparrow) = \varphi(\uparrow)\varphi(\uparrow)$ ? • Behaviour under disk unzip.

Equations for Vanilla / Unitary V.

- ✓1. Unitarity  $V(V//dA^{12}) = 1$
- ✓2. VF4V  $V(V//dS^{12}) = R$
- ✓3. Cap  $VC_{12} = C_1C_2$  in  $\mathcal{A}(\uparrow_{12})$
- 4.  $V//dS^2//dm_1^{12} = C^{-2}(R_{11}^{-1/2}//dS^1)$
- 5.  $V//h\eta^2//dS^1//dm_1^{21} = C^{-2}$
- 6.  $V//h\eta^1//dS^2//dm_1^{12} = ((R_{11}^{1/2}//dS^1)C^2)^{-1}$
- ✓7.  $VV_{\otimes} = C_1^{-1}C_2^{-1}V_{\otimes}C_{12}$

Equations with Simple Buckle.

- ✓b 0.  $V_b := C_1C_2VC_{12}^{-1}$
- ✓b 1. Unitarity  $V_bC_{12}^2(V_b//dA^{12}) = C_1^2C_2^2$
- ✓b 2. VF4V  $V_bC_{12}^2(V_b//dS^{12}) = RC_1^2C_2^2$
- ✓b 3. Cap  $V_bC_{12}^2 = C_1^2C_2^2$  in  $\mathcal{A}(\uparrow_{12})$
- 4.  $V_b//dS^1//dm_1^{21} = R_{11}^{-1/2}$
- 5.  $V_b//h\eta^2//dS^1//dm_1^{21} = 1$
- 6.  $V_b//h\eta^1//dS^2//dm_1^{12} = (R_{11}^{1/2}//dS^1)^{-1}$
- ✓b 7.  $V_bV_{\otimes} = V_{\otimes}$

Equations with Trivial Caps.

- 0.  $V_c := C_{12}VC_1^{-1}C_2^{-1}$
- 1. Unitarity  $V_cC_{12}^{-2}(V_c//dA^{12}) = C_1^{-2}C_2^{-2}$
- 2. VF4V  $V_cC_{12}^{-2}(V_c//dS^{12}) = RC_1^{-2}C_2^{-2}$
- 3. Cap  $V_c = 1$  in  $\mathcal{A}(\uparrow_{12})$
- 4.  $V_c//dS^2//dm_1^{12} = C^{-4}(R_{11}^{-1/2}//dS^1)$
- 5.  $V_c//h\eta^2//dS^1//dm_1^{21} = C^{-4}$
- 6.  $V_c//h\eta^1//dS^2//dm_1^{12} = ((R_{11}^{1/2}//dS^1)C^4)^{-1}$
- 7.  $V_cV_{\otimes} = C_1^{-2}C_2^{-2}C_{12}^2V_{\otimes}$

