Kashaev: Une nouvelle formulation de la TQFT de Teichmuller

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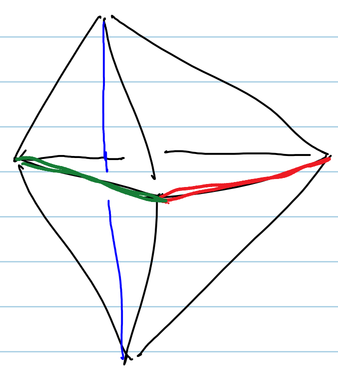
8:10 AM

statistical mechanical model on "shaped triangulations".

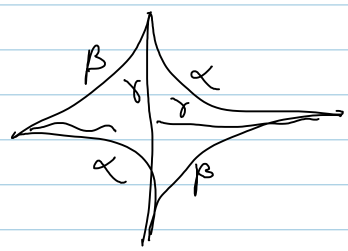
Triangulation := Δ-triangulations in the sense of Hatcher's book – a cellular complex in which all cells are standard simplices, with standard gluing maps [all vertices of cells ordered].

Shaped := Every tetrahedron carries dihedral angles of an ideal hyperbolic tetrahedra.

Shaped 2-3 pachner move:



ideal tetrahedra:



$\alpha+\beta+\gamma=\pi$

shaped move: keep the same sum of angles on corresponding edges.
total dihedral angle along central edge is 2π.

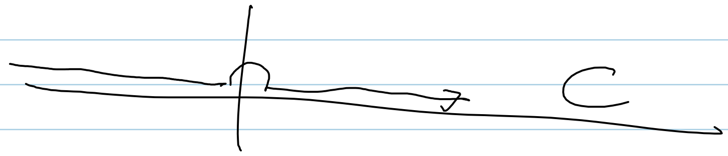
Problem: Given a shaped triangulation X. Find an invariant $f(X)$ under shaped Pachner moves.

State space: X: Triangulation. $\Delta_1(X)$: The set of edges.

$$\| \quad [0,1]^{\Delta_1(X)}$$

Special function: Faddeev's quantum dilogarithm

$$\Phi_\hbar(z) = \exp\left(\frac{1}{4}\int_C \frac{e^{-2ixz}}{\sinh(bx)\sinh(b^{-1}x)}\,\frac{dx}{x}\right)$$



$$(b+b^{-1})^2 = \hbar^{-1}$$

if $\operatorname{im} b > 0$, $\Phi_\hbar(z) = \dfrac{(e^{2\pi b(z+c_b)};q)_\infty}{\cdots}$

$(\hbar < \frac{1}{4})$

So function is meromorphic w/ all poles & zeros explicitly known.

Satisfies a pentagon identity:

$$\Phi_h(\hat{p})\,\Phi_h(\hat{q}) = \Phi_h(\hat{q})\,\Phi_h(\hat{p}+\hat{q})\,\Phi_h(\hat{p})$$

where $\hat{p}\hat{q}-\hat{q}\hat{p} = \frac{1}{2\pi i}$

Weil-Gel'fand-Zak transformation:

$f: \mathbb{R} \longrightarrow \mathbb{C}$ Schwartz function

$$W(f)(x,y) = e^{\pi i x y} \sum_{m\in\mathbb{Z}} f(x+m)\, e^{2\pi i m y}$$

$$W(f)(x,y+1) = e^{\pi i x} W(f)(x,y)$$

$$W(f)(x+1,y) = e^{-\pi i y} W(f)(x,y)$$

So $W(f)$ is a section of a line bundle over $\mathbb{T}^2$.

$$W^{-1}(g)(x) = \int_0^1 g(x,y)\, e^{-\pi i x y}\, dy$$

w/ Fourier $\mathcal{F}$,

$$(W\mathcal{F}f)(x,y) = (Wf)(-y,x)$$

45 mins.

Next come $g_{a,c}(x)$, Boltzmann weights, partition function, invariance theorem.

Why interesting?

Conjecture:

$$2\pi\hbar \log\left|-Z_{1/\hbar}(X)\right| \xrightarrow{\hbar\to 0} -\mathrm{Vol}(X)$$

Proved for some X's.