Geneva October 24 talk on Finite Type Invariants of Ribbon Knotted Balloons and Hoops

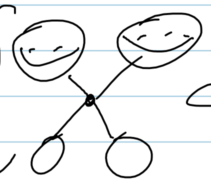
October-08-13
8:20 AM

Abstract. On my September 17 Geneva talk I described a certain trees-and-wheels-valued invariant of ribbon knotted loops and 2-spheres in 4-space, and my October 8 Geneva talk describes its reduction to the Alexander polynomial. Today I will explain how that same invariant arises completely naturally within the theory of finite type invariants of ribbon knotted loops and 2-spheres in 4-space.
This talk will be self-contained and the only prerequisites for it are some basic linear algebra and having no fear of exponentials.

Pasted from <http://www.math.toronto.edu/~drorbn/Talks/Geneva-131024/>

Content:

1. Flash



$$Z:\overbrace{\left\{\quad \hookrightarrow \mathbb{R}^4\right\}}^{=K^{bh}} \longrightarrow \underbrace{FL(T)}_{\text{"trees"}}{}^{H} \times \underbrace{CW(T)}_{\text{"wheels"}}$$

my goal is to tell you why such invariant is expected, not to derive the formulas.

2. $K^{bh} = \mathbb{Q}\{\text{Pictures}\}/\text{relations.}$

Disturbing conjecture: That's all.

3. $\mathcal{I}^n = \{\text{Picture w/ sing.}\} \subset K^{bh}$

Want "expansion" $Z: K^{bh} \to \mathcal{A}^{bh} := \widehat{\bigoplus} \mathcal{I}^n/\mathcal{I}^{n+1}$

s.t. if $\gamma \in \mathcal{I}^n$, $Z(\gamma) = (0, \ldots 0, \gamma/\mathcal{I}^{n+1}, *, * \ldots)$ "property U"

4. Why?

a. Just because, and this is vastly more general.

b. $\left(K^{bh}/I^{n+1}\right)^*$ is "finite type/polynomial invariants"

c. The Taylor example.

5. I'll construct

$$\begin{tikzcd}
 & \tilde{\mathcal{A}}^{bh} \arrow[d, "\pi"] \\
K^{bh} \arrow[ur, "\tilde{Z}"] \arrow[r, "Z"'] & \mathcal{A}^{bh} = \bigoplus I^n/I^{n+1}
\end{tikzcd}$$

Definition 4.7. A "candidate projectivization" for an algebraic structure $\mathcal{O}$ is a graded structure $\mathcal{A}$ with the same operations as $\mathcal{O}$ along with a homomorphic surjective graded map $\pi\colon \mathcal{A} \to \operatorname{proj}\mathcal{O}$. An "$\mathcal{A}$-expansion" is a kind and filtration respecting map $Z_{\mathcal{A}}\colon \mathcal{O} \to \mathcal{A}$ for which $(\operatorname{gr} Z_{\mathcal{A}}) \circ \pi\colon \mathcal{A} \to \mathcal{A}$ is the identity. There's no need to define "homomorphic $\mathcal{A}$-expansions".

$$\begin{tikzcd}
 & \mathcal{A} \arrow[d, shift right, "\pi"'] \\
\mathcal{O} \arrow[ur, "Z_{\mathcal{A}}"] \arrow[r, "Z"'] & \operatorname{proj}\mathcal{O} \arrow[u, shift right, "\operatorname{gr} Z_{\mathcal{A}}"']
\end{tikzcd}$$

1. π is surjective.
2. $\pi /\!/ \operatorname{gr}\tilde{Z} = Id$

$\Longrightarrow$ 1. Z is an expansion
2. π is an isomorphism.

6. Describe $\tilde{\mathcal{A}}^{bh}$

7. Describe $\tilde{Z}$ [Warning: Not easily computable!]

8. Bracket rise.

9. Trees and wheels, $\zeta = \log Z$.