I would prefer to call it an "atlas" or a dictionary". The word "table" suggests simple enumeration; the word "atlas" suggests a wealth of further information.

Maybe a "repository"?

BUILDING A TABLE OF TANGLES

KEN BAKER, NEIL HOFFMAN, DROR BAR-NATAN, AND ANASTASIIA TSVIETKOVA

The fields of topology and geometry have been greatly advanced by a wealth of simple examples to analyze. In the study of 3-manifolds from topological, geometric, combinatorial and quantum points of view, the first examples that allow to check intuitive ideas are often knots and links. The purpose of this research project is to extend the classification of knots and links to a classification of tangles, which one can consider as building blocks for knots and links.

While *tangle* can describe a variety of objects, we will consider tangles comprised of a arcs and b circles embedded in the closed 3-ball such that the a arcs have their endpoints on the boundary of the 3-ball. First, we seek to enumerate all tangles such that $a = 2$ of small complexity, namely that the tangle admits a planar diagram having m or fewer crossings. (The actual m will be determined later by the computation time of our algorithms.) Second, we will classify these tangles up to ~~homeomorphism~~ isotopy type. In future work, we would seek to develop and apply techniques to higher values for a.

The results of this project should be of interest to (at least) two audiences. First, ~~3-manifold topologists~~ knot theorists will benefit from having this "infrastructure project" addressed. By providing a concise and defined table to test ideas against, topologists will be able to understand their relevance and to gauge their computability of any new tangle invariant. There are numerous papers where this analysis has been applied to the tabulations of knots and links (see papers citing [4] and Rolfsen's table [6]). Tangles themselves form the building blocks for knots and links, and so a deeper understanding of tangles would lead to advances in the field. A second audience for this project is applied mathematicians, such as, for example, DNA topologists. DNA is naturally tangled in the cell, however, the topology of the DNA as an embedded tangle is one factor that affects how a cell can replicate as well as a variety of other interesting behavior (see [2] for example). Often, DNA topologists are interested in how a tangle behaves under *tangle filling*, i.e. closing of a tangle with a arcs embedded in a 3-ball with a second tangle having a arcs in a 3-ball.

Our research program is straightforward, however we will now provide greater detail to what is mentioned above. First, we seek to extend and implement the enumeration of knots and links as performed in the tables of [1] to our setting in order to enumerate all possible tangles with small crossing number comprised of two arcs

and n circles. Second, we hope to classify all such tangles up to homeomorphism. Initially, there are two techniques we hope to use to distinguish these tangles. In her thesis (see [5, §4.2]), the fourth investigator applies the idea of *encircled tangles*, namely adding an extra embedded circle to a tangle diagram such this embedded circle alternates between over and under crossings with the two arcs of the tangle that connect to the boundary. Excluding a few pathological cases, the tangle inside the circle has a canonical hyperbolic structure independent of the link containing it by [5, Theorem 4.2.3].

Additionally, to any tangle (B, T), we can associate a double branched cover M such that there is a continuous map $f : M \to (B, T)$ that is two-to-one between $M - f^{-1}(T)$ and $B - T$ and one-to-one on $f^{-1}(T)$ and T. This double branched cover will be a 3-manifold, and so invariants of M can be applied in distinguishing the homeomorphism type of (B, T). Furthermore, in the case that T has only two arcs M will have a torus boundary (see [7] for example) and so more refined geometric invariants such has hyperbolic volume associated to M can be employed as part of the identification. While some ideas along these lines have been implemented as part of the software ORB [3], that piece of software is no longer being updated. In addition to the updates for compatibility with current computer architectures, this software would also need to be tweaked to include a scriptable interface.

The SQuaREs model is well suited for building such a table of tangles. This project will require a considerable use of computational tools. Having a number of people working in collaboration to first build and then later maintain and grow such a table seems ideal for the work that needs to be done. In addition, the final product will be freely available software that would include a sortable table similar to [1]. Thus, drawing on the ideas and inputs from each of four authors will ultimately lead to a more flexible interface that will provide maximum utility to the user. Since our audience is both 3-manifold topologists and biologist involved understanding applied problems relating to DNA, ending with the most user friendly implementation is paramount.

In addition, our team meets the criterion of the SQuaREs model. This collaboration would bring together not only four people at different stages of their careers, namely two post doctoral researchers and two professors, but also four people that are well suited to solve this problem because of their variety of expertise.

Please forgive me - I think such a project, if done right, will be *exteremely* valuable. Yet due to other commitments I cannot be a very active participant. I don't want to feel that I am making a promise to AIM that I don't intent to keep, so I'd appreciate it if you could add to the proposal a sentence in the spirit of the sentence below:

> The primary researchers in this project will be A, B, and C; DBN, who initiated and maintains a similar project, "The Knot Atlas", will be taking an advisory role.

References

[1] Dror Bar-Natan et. al, The knot atlas, `http://katlas.org`.
[2] Dorothy Buck and Eric Flapan *Applications of Knot Theory* , AMS, 2009.
[3] Damian Heard, Orb, based on Jeff Weeks' Snappea kernel, `http://www.ms.unimelb.edu.au/~snap/orb.html`.
[4] Jim Hoste and Morwen Thistlethwaite and Jeff Weeks *The first 1,701,936 knots* The Mathematical Intelligencer, 20(4) 33-48, 1998.
[5] Anastasiia Tsvietkova *Hyperbolic Structures from Link Diagrams*, ProQuest LLC 2012.
[6] Dale Rolfsen *Knots and lins*, AMS Chelsea Pub. 2003.
[7] Genevieve Walsh *Orbifolds and commensurabilty*, "Interactions Between Hyperbolic Geometry, Quantum Topology and Number Theory" Contemporary Mathematics Vol 541 221–231, (2011).

Ken Baker, Assistant Professor, Department of Mathematics, Univeristy of Miami, Miami, FL

E-mail address: kenken@math.miami.edu

Neil Hoffman, Post doctoral researcher, Department of Mathematics and Statistics, University of Melbourne, Melbourne, Australia

E-mail address: nhoffman@ms.unimelb.edu.au

Dror Bar-Natan, Professor, Department of Mathematics, University of Toronto, Toronto, Ontario, Canada

E-mail address: drorbn@math.toronto.edu

Anastasiia Tsvietkova, Post doctoral researcher, ICERM, Brown University, Providence, RI

E-mail address: n.tsvet@gmail.com

My main comment may be a bit hard to implement so late, yet we should be aware of it if we are to work on this project. Our project shouldn't be merely a "table", but also and maybe mostly a web- and a programming- interface to that table. On the web side, we should have a friendly page for each tangles with pictures and invariants and symmetries and comments and hot links to "neighboring" tangles that are obtained by adding a single crossing along the edge or by performing other basic operations. And there should be an API for all that.