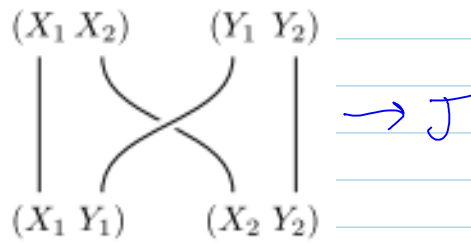


The next-simplest example

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4:36 AM

Pavol writes: For the next simplest example look in my paper with David, sect.3. You can read it, but you can just look at the parenthesized braid with 4 strands which is drawn there. Let J in $U(\mathfrak{g}+\mathfrak{g}+\mathfrak{g})[[\hbar]]$ be what corresponds to this braid.



Now if A is an algebra in $U(\mathfrak{g}+\mathfrak{g})\text{-Mod}^\Phi$ (notice: there is no bar over any \mathfrak{g}) then:

S (S for swinging)
perhaps I should make a "properties of S" mini-poster.

1. A is a $U\mathfrak{g}$ -module, where this \mathfrak{g} is the diagonal in $\mathfrak{g}+\mathfrak{g}$, but the original product m on A is no longer associative in $U\mathfrak{g}\text{-Mod}^\Phi$

2. however, if we compose m with the action of $J: A \otimes A \rightarrow A \otimes A$, then this new product m' is associative in $U\mathfrak{g}\text{-Mod}^\Phi$

A. This is really $U(\mathfrak{g}) \otimes U(\mathfrak{g})\text{-Mod}$

3. we can have there a spectator Lie algebra \mathfrak{h} , i.e. A is in $U(\mathfrak{g}+\mathfrak{g}+\mathfrak{h})\text{-Mod}^\Phi$, and we make it to an associative algebra in $U(\mathfrak{g}+\mathfrak{h})\text{-Mod}^\Phi$ (with the same J in $U(\mathfrak{g}+\mathfrak{g}+\mathfrak{g})$)

This I must understand
...and the problem is that I don't understand what is "an algebra"

There are now two possibilities for the "next simplest" algebra:

1. start with $A = C^\infty((G \times \bar{G})/G \times (G \times \bar{G})/G) = C^\infty(G \times G)$, which is commutative associative in $U(\mathfrak{g}+\mathfrak{g}+\bar{\mathfrak{g}})$, treat $\bar{\mathfrak{g}}$ as the spectator \mathfrak{h} ; the composition of the original product with the action of J makes A to an associative (but not commutative) algebra in $U(\mathfrak{g}+\bar{\mathfrak{g}})$. This is the quantization of the moduli space of a triangle (with one vertex marked with + and two with -)

2. if spectator Lie algebras don't sound so simple, choose any coisotropic $C_1, C_2 \subset G$, and start with $C^\infty(G/C_1 \times G/C_2)$ as a commutative associative algebra in $U(\mathfrak{g}+\mathfrak{g})\text{-Mod}^\Phi$.