Cheat Sheet 3D Topology

Material from Hatcher's notes and from Hempel's book.

http://drorbn.net/AcademicPensieve/2013-08/
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Theorem (Alexander, 1920s). An embedded 2-sphere in $\mathbb{R}^3$ bounds a 3-ball. $(\Rightarrow S^3$ is irreducible$)$

A

Dehn's Lemma (Dehn 1910 (wrong), Papakyriakopoulos 1950s). M a 3-manifold, $f\colon B^2 \to M$ s.t. for some neighborhood A of ∂B^2 in B^2 the restriction $F\mid_A$ is an embedding and $f^{-1}(f(A)) = A$. Then $f\mid_{\partial B^2}$ extends to an embedding $g\colon B^2 \to M$.

The Loop Theorem (Stallings 1960, implies Dehn's lemma). M a 3-manifold, F a connected 2-manifold in ∂M, $\ker(\pi_1(F) \to \pi_1(M) \not\subset N \triangleleft \pi_1(F)$. Then there is a proper embedding $g\colon (B^2, \partial B^2) \to (M, F)$ s.t. $[g\mid_{\partial B^2}] \notin N$.

The Sphere Theorem. M orientable 3-manifold, N a $\pi_1(M)$-invariant proper subgroup of $\pi_2(M)$. Then there is an embedding $g\colon S^2 \to M$ s.t. $[g] \notin N$.

A: Prime: $M = P \# Q \Rightarrow (P = S^3) \vee (Q = S^3)$

Irreducible: An embedded $\underbrace{S^2}_{\text{in } M}$ bounds a B^3.

<u>Thm</u> M orientable, prime, not irreducible

$\Rightarrow M = S^2 \times S^1$

To do. • Schoenlies, JSJ.