Cheat Sheet 3D Topology

Material from Hatcher's notes and from Hempel's book.

http://drorbn.net/AcademicPensieve/2013-08/

initiated 31/8/13; modified 8/9/13, 8:38pm; continued 2013-09

Theorem (Alexander, 1920s). An embedded 2-sphere in $\mathbb{R}^3$ bounds a 3-ball.

Dehn's Lemma (Dehn 1910 (wrong), Papakyriakopoulos 1950s). M a 3-manifold, $f\colon B^2 \to M$ s.t. for some neighborhood A of ∂B^2 in B^2 the restriction $F\,|_A$ is an embedding and $f^{-1}(f(A)) = A$. Then $f\,|_{\partial B^2}$ extends to an embedding $g\colon B^2 \to M$.

The Loop Theorem (Stallings 1960, implies Dehn's lemma). M a 3-manifold, F a connected 2-manifold in ∂M, $\ker(\pi_1(F) \to \pi_1(M) \not\subset N \triangleleft \pi_1(F)$. Then there is a proper embedding $g\colon (B^2, \partial B^2) \to (M, F)$ s.t. $[g\,|_{\partial B^2}] \notin N$.

The Sphere Theorem. M orientable 3-manifold, N a $\pi_1(M)$-invariant proper subgroup of $\pi_2(M)$. Then there is an embedding $g\colon S^2 \to M$ s.t. $[g] \notin N$.

To do. • Schoenlies, JSJ.