Grigsby: Plamenevskayas' invariant and trivial braid detection

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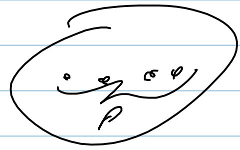
9:30 AM

Joint w/ J. Baldwin & S. Wehrli

$L \subset S^3 \to Kh(L)$ a bigraded vector space over $\mathbb{F} = \mathbb{Z}/2$.

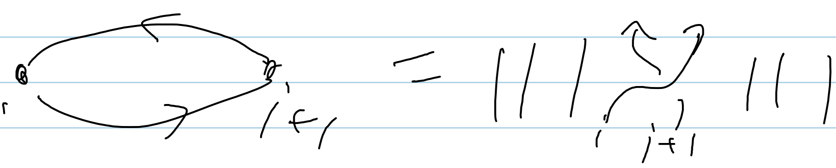
An application to braids w/ a <u>completely</u> <u>combinatorial</u> proof.

B_n = n-strand braid group

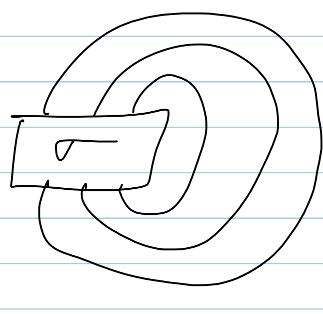
$= \mathrm{Diff}^+(D_n =$ , ∂D, p $)$ / isos

$p = \{1 \dots n\}$ — ∂D pointwise fixed — p setwise fixed

Artin generators $\sigma_1 \dots \sigma_{n-1}$

$\sigma_i :$ 

$\mathbb{1} = ||||$

$\sigma \longmapsto \text{closure}(\sigma) = \hat{\sigma}$ 

<u>Thm A</u> (originally due to Birman

& Menasco using braid foliations)
(G-Wehrli using Kh)

Let $\sigma \in B_n$. If $\hat{\sigma}$ is the trivial n-link, then $\sigma = \mathbb{1}$.

Original B-M proof: Corollary of "monotone simplification" Thm for braids representing the unlink.

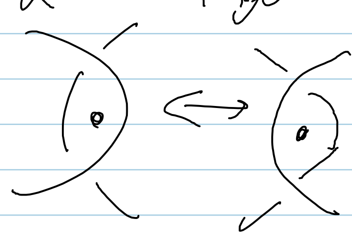
Markov Thm: If $\hat{\sigma} = \hat{\sigma'}$, then they differ by isotopies, conjugation & stabilization / de-stabilizations.

MT w/o stabilization:

$$\underset{B_M}{\hat{\sigma}} = \underset{B_m}{\hat{\sigma'}} = \underset{\text{n-unlink}}{U_n} \qquad M > m > n$$

[Example due to Morton of a 4-strand braid representing the unknot for which de-stab is necessary]

$\Rightarrow \sigma \longrightarrow \sigma'$ using conjugations, destabs, & exchange moves:

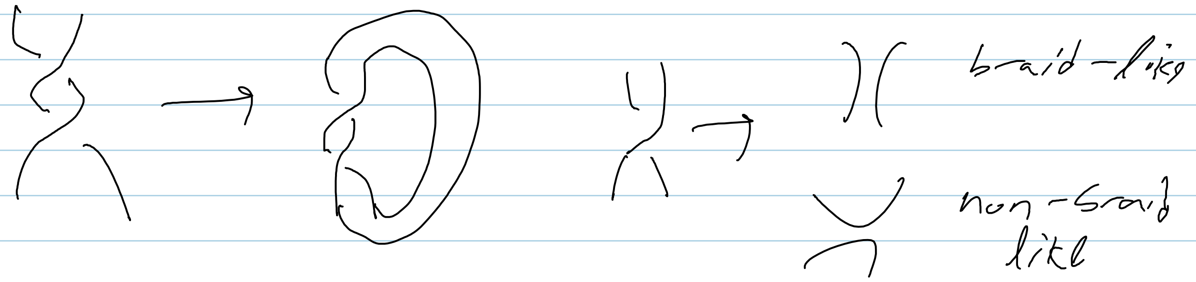


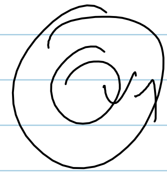
New proof using KH:

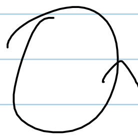
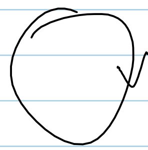
Outside input: "Dehornoy order"

on B_n

A bit on KH & Plamenev.. invariant:



Get a cube of resolutions, with generators "oriented Kauffman states"  etc.

 $\longleftrightarrow v_+ = 1$  $\longleftrightarrow v_- = x$

.... there are boundary maps.

Plamen.: There is a distinguished cycle in this complex: The "all clockwise" generator on the "all-braidlike" resolution call it $\psi(\sigma)$. $\Psi(\sigma) = [\psi(\sigma)] \in KH$

Plam also showed that $\Psi(\sigma)$ is an invt. of transverse isotopy class of σ, in particular, it's conjugacy class.

Thm B (G-Baldwin) Let $\sigma \in B_n$, $m(\sigma)$ its mirror. Then if $\psi(\sigma) \neq 0$ & $\psi(m(\sigma)) \neq 0$, then $\sigma = \mathbb{1}$.

Remark: Thm B & proof is inspired by observation of Hedden - Van Horn-Morris - Watson: ...... Something about fibred & monodromy & contact.....

Follows from HF stuff...

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