CUMC Handout on July 6, 2013 (1)

July-06-13
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Dror Bar-Natan: Talks: CUMC-1307:
ω :=http://www.math.toronto.edu/~drorbn/Talks/CUMC-1307

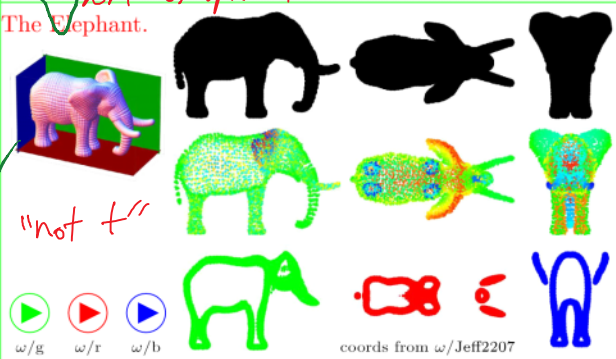
Visualizing the Fourth Dimension

with Ester Dalvit
ω/Dal

Abstract. Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and this achieved, I will tell you about the simplest problem in 4-dimensional knot theory whose solution I don't know.

"Flatlanders view an elephant"

The Elephant.



ω/g ω/r ω/b

coords from ω/Jeff2207

"not t"

Knots.

ω/P
ω/M1
ω/M2

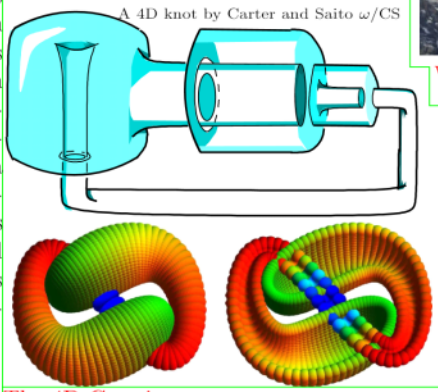
Do one knot "right"

1. Project and hide.
2. Project and colour code.
3. Slice and animate.

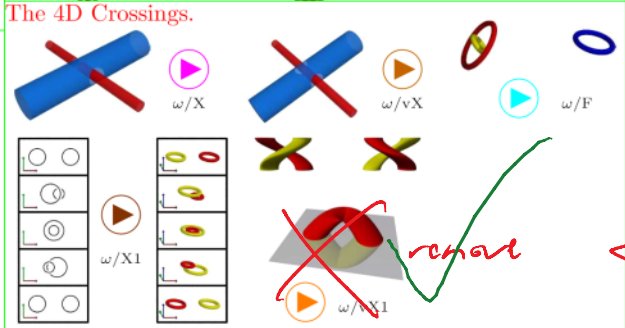
The Crossings.

4D Knots.

A 4D knot by Carter and Saito ω/CS



The 4D Crossings.



ω/X ω/vX ω/F

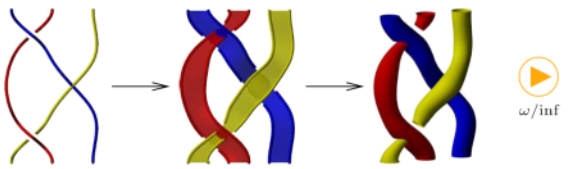
ω/X1

remove

ω/vX1

View as flying rings.
View as coloured tubes.
View as cut tubes
View as "inflated bands".

The Inflation Procedure.



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and the Simplest Thing I Don't Know About It

Satoh's Conjecture. The "kernel" of the "double inflation" map δ, mapping "long" w-knot diagrams in the plane to "long" knotted 2D tubes in 4D, is precisely the moves R1–R3, VR1–VR3, D and OC listed below.


Shin Satoh

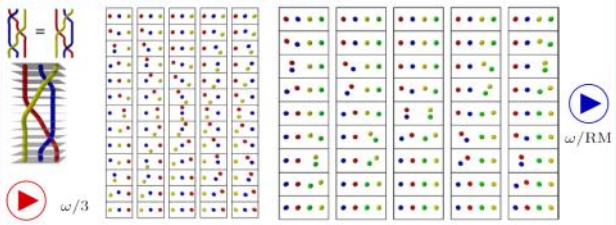
In other words, two long w-knot diagrams represent via δ the same long 2D knotted tube in 4D iff they differ by a sequence of the said moves.

First Iso. Thm: $\phi\colon G \to H \;\Rightarrow\; \operatorname{im}\phi \cong G/\ker(\phi)$

δ is a map from algebra to topology. So a thing in "hard" topology ("ribbon 2-knots") is the same as a thing in "easy" algebra.

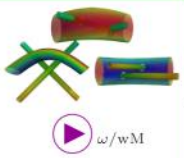
What's "The Same"?

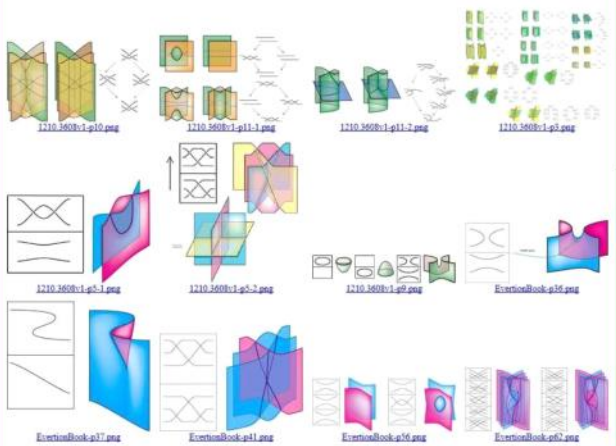
Reidemeister Moves and Theorem.



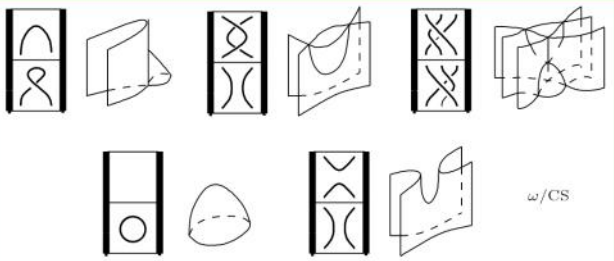
w-Moves.


Carter, Banach, Saito





Roseman Moves.



Add the Roseman Picture

Movie Moves.