BF in Degree 1 from 0210037v1 - Cattaneo Rossi - Wilson Surfaces

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This would make an excellent graduate project!

6.1. **Order 1.** The only possible term at order 1 has the form $\Theta_1 \operatorname{Tr}(\operatorname{ad} \Xi)$ with

$$\Theta_1 = \int_{C_{2,0}} \theta_{12}\, \eta_{12}. \tag{6.2}$$

Observe that this term does not appear if $\mathfrak{g}$ is unimodular. It is also possible to prove (considering the involution $(x_1, x_2) \mapsto (x_2, x_1)$ of $C_{2,0}$) that Θ_1 vanishes if m is odd. The graphical representation of Θ_1 is displayed in fig. 7. (From now on we omit in diagrams the black and white strip representing Ξ. In fig. 7 it would be attached to vertex 1.)

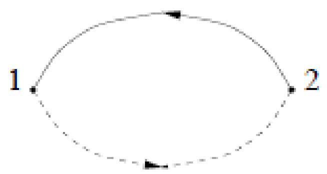

FIGURE 7. Order 1.

In even dimensions, Θ_1 furnishes a function on Imb_σ which is a generalization of the self-linking number for ordinary knots. *This function is not an invariant.* It can be easily proved that, in computing the differential of Θ_1, the only boundary contribution corresponds to the collapse of the two points. One obtains then

$$\mathrm{d}\Theta_1 = -p_{1*}(\Phi^* w_{m-1} \wedge p_3^* w_{m-3}),$$

where

$$\begin{array}{rccc} \Phi\colon & \mathrm{Imb}_\sigma \times \mathbb{R}^{m-2} \times S^{m-3} & \to & S^{m-1} \\ & (f, x, v) & \mapsto & \dfrac{\mathrm{d}f(x)v}{\|\mathrm{d}f(x)v\|} \end{array}$$

and p_i denotes the projection to the ith factor.[16]

$Y : \mathbb{R}^2 \to \mathbb{R}^4$ "long"

Get "$g\gamma$" (the Gauss map of γ) —

$$g\gamma : \underbrace{\mathbb{R}^2 \times S^1}_{\partial \overline{C}_2 = T_{unit}\mathbb{R}^2} \longrightarrow S^3 \times S^1 \qquad (d\gamma, P_2)$$

A 2-parameter family $\overbrace{\text{of circles}}^{\text{"grant"}}$ in S^3

$\pi_3(gr_2(\mathbb{R}^4))$

$$\begin{pmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & a \\ 0 & 0 & 1 & * \end{pmatrix}$$

$$\begin{pmatrix} 1 & * & 0 & * \\ 0 & 0 & 1 & * \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & a & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & * & * & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & a \\ 0 & 0 & 1 & a \end{pmatrix} \to \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & a \end{pmatrix} \to \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$gr_1 \mathbb{R}^3 \sim S^2$