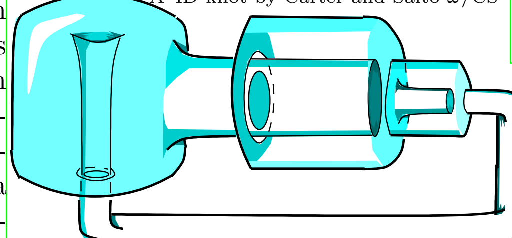
Visualizing the Fourth Dimension

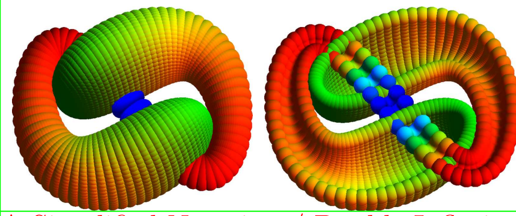
with Ester Dalvit
ω/Dal

Abstract. Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and this achieved, I will tell you about the simplest problem in 4-dimensional knot theory whose solution I don't know.

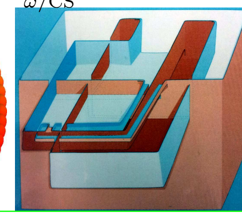
4D Knots.

A 4D knot by Carter and Saito ω/CS

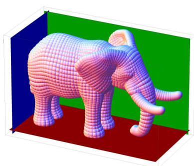




ω/CS

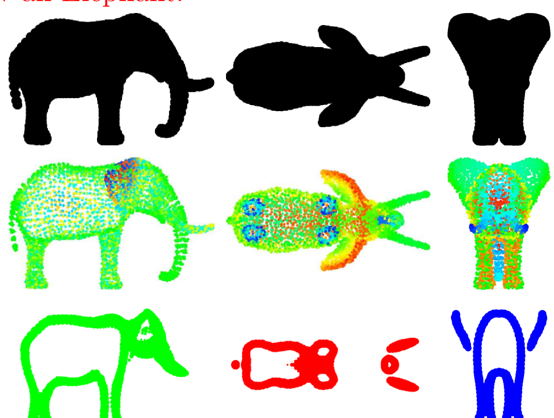


Flatlanders View an Elephant.



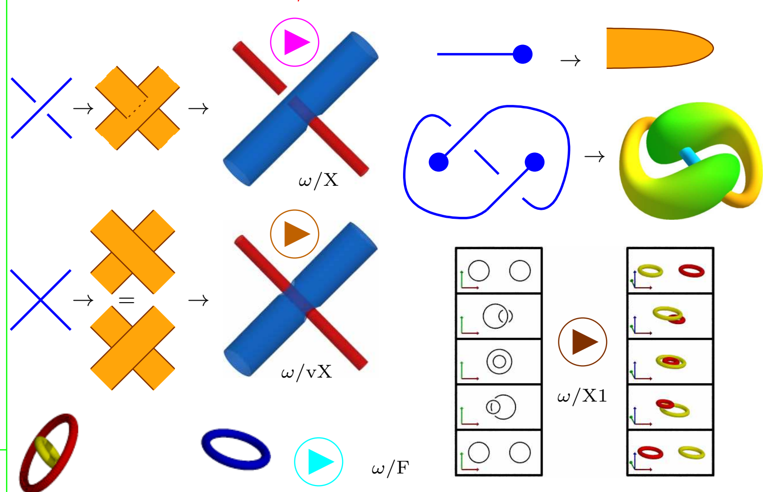
"The third dimension isn't t"

ω/g ω/r ω/b

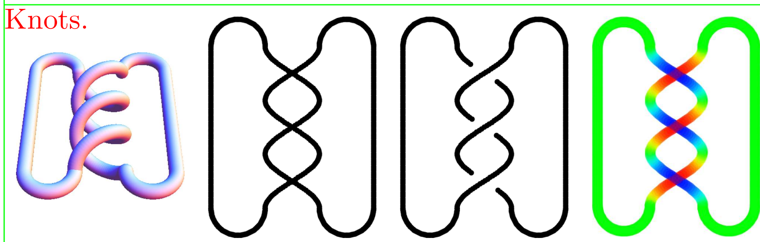


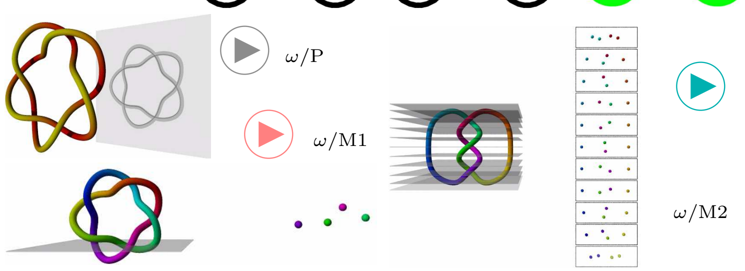
coords from ω/Jeff2207

A Simplified Notation / Double Inflation

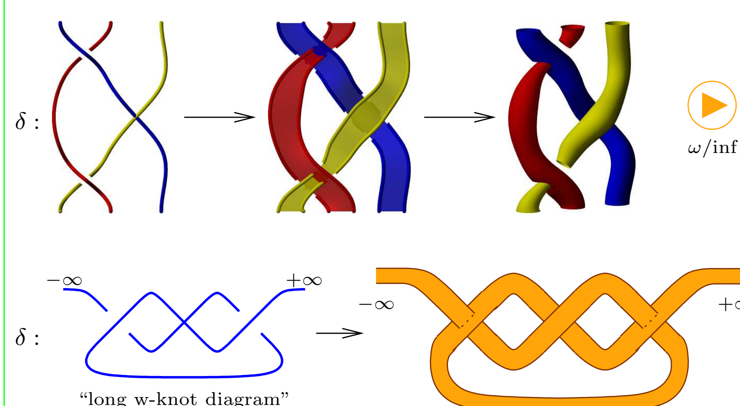


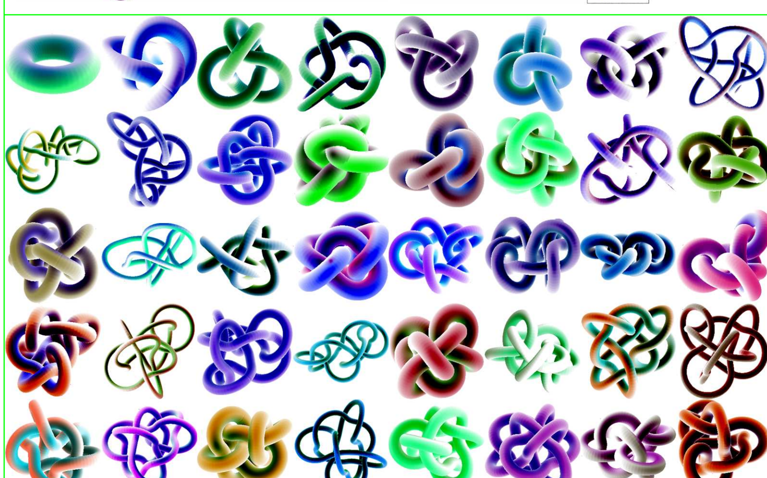
Knots.



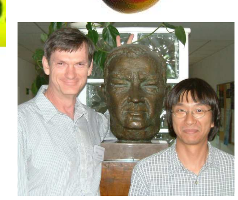


The Double Inflation Procedure δ.





Many of the images are by Carter and Carter-Saito, ω/CS.



Carter, Banach, Saito

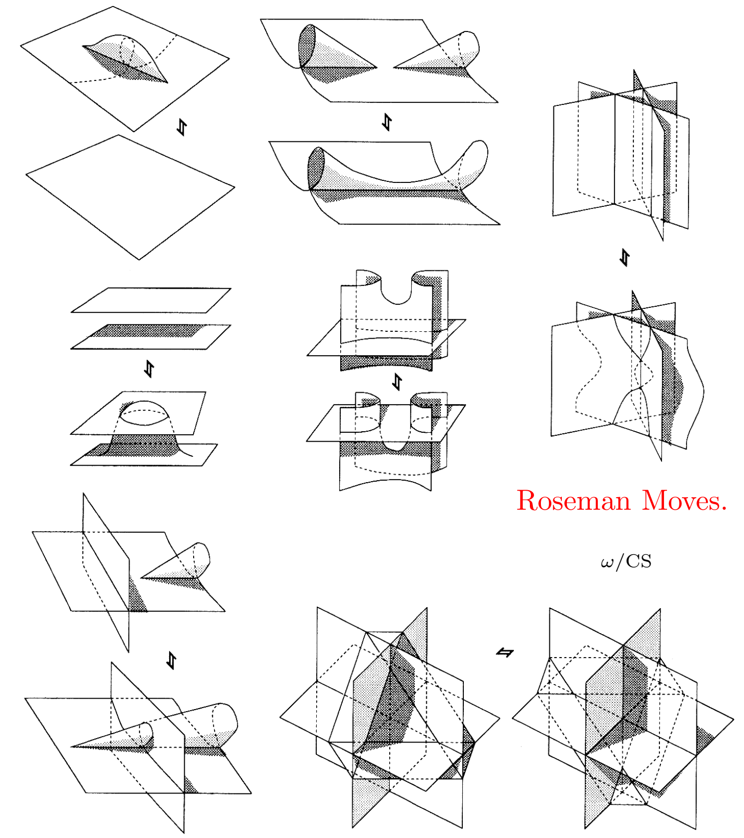
and the Simplest Thing I Don't Know About It


Shin Satoh

Satoh's Conjecture. (ω/Sat) The "kernel" of the "double inflation" map δ, mapping "long" w-knot diagrams in the plane to "long" knotted 2D tubes in 4D, is precisely the moves R1–R3, VR1–VR3, D and OC listed below. In other words, two long w-knot diagrams represent via δ the same long 2D knotted tube in 4D iff they differ by a sequence of the said moves.

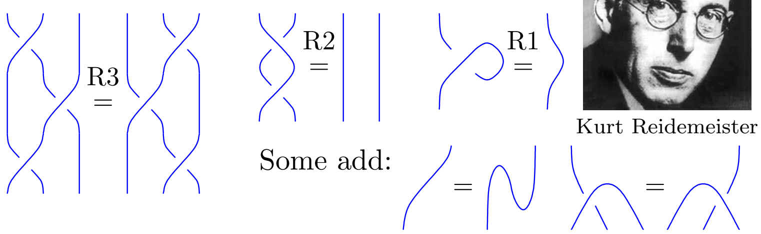
$$\text{First Iso. Thm: } \phi\colon G \to H \;\Rightarrow\; \operatorname{im}\phi \cong G/\ker(\phi)$$

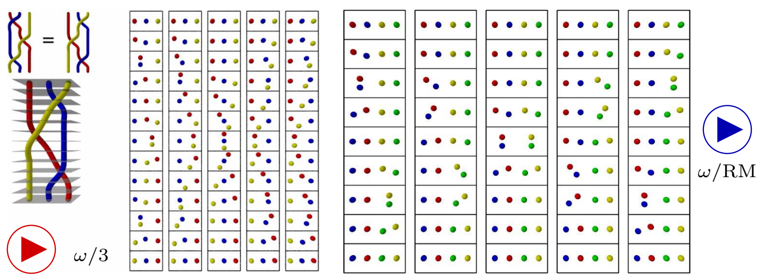
δ is a map from algebra to topology. So a thing in "hard" topology ("ribbon 2-knots") is the same as a thing in "easy" algebra. What's "The Same"?

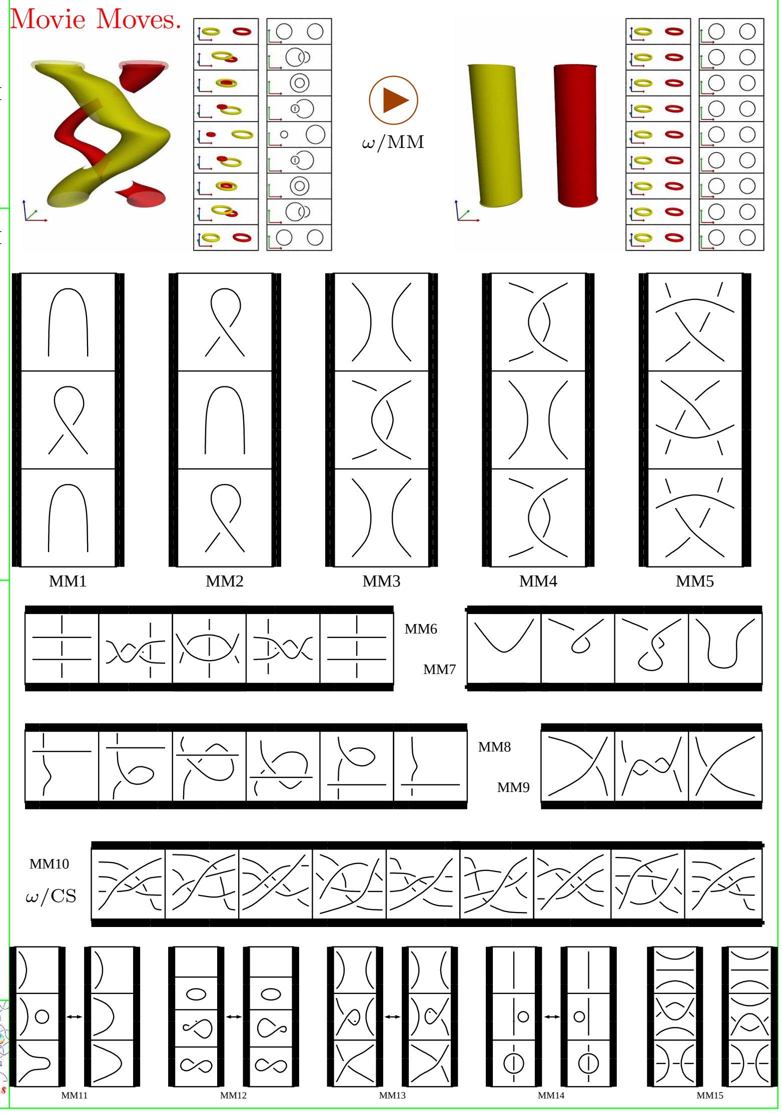



Kurt Reidemeister

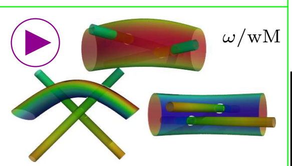
Reidemeister' Theorem. Two knot diagrams represent the same 3D knot iff they differ by a sequence of "Reidemester moves":

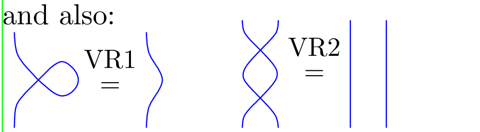


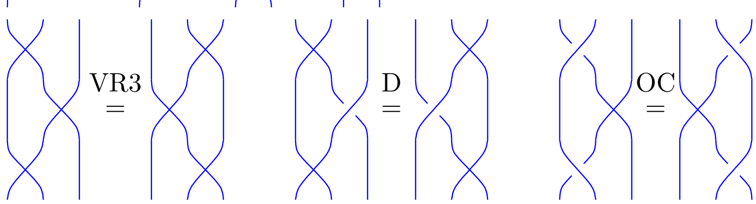


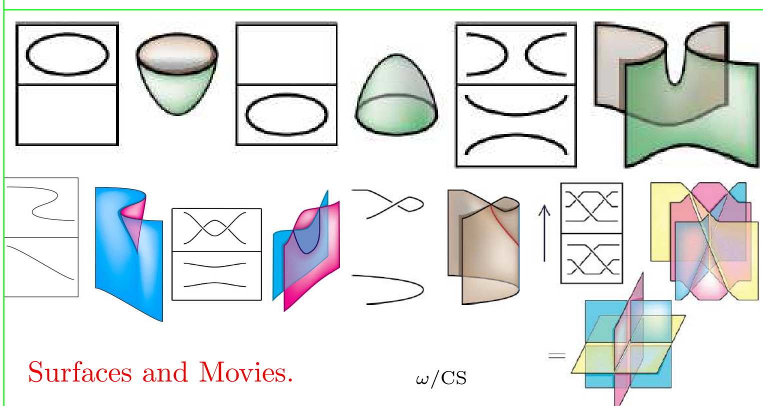


w-Moves. Same R1, R2, R3 as above, and also:









Surfaces and Movies.

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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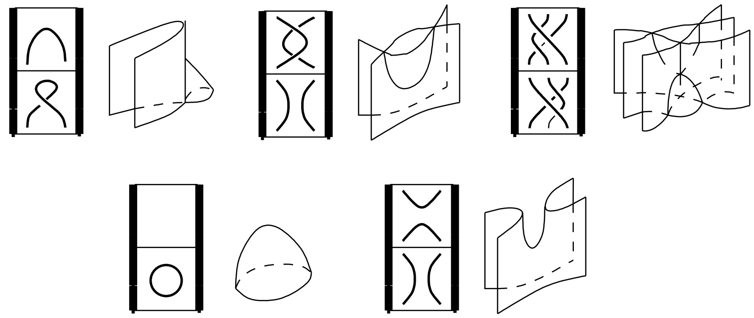
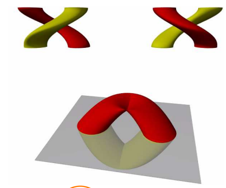
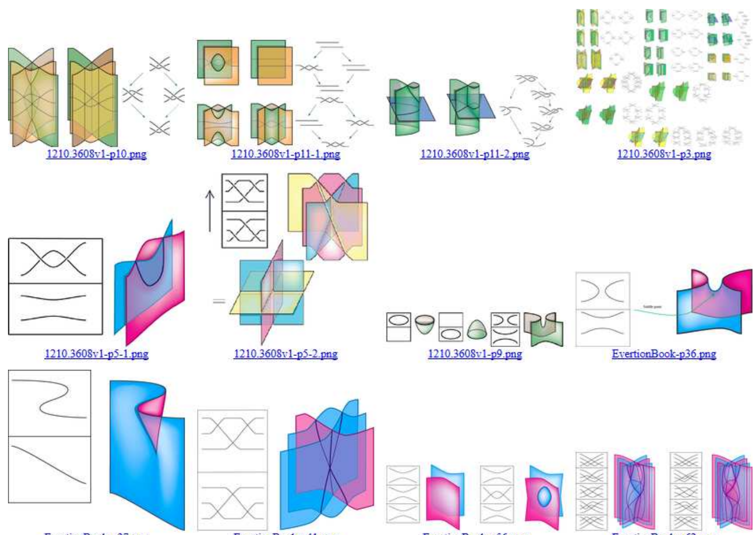
Recycling





ω/vX1

ω/S



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EvertionBook-p37.png   EvertionBook-p41.png   EvertionBook-p56.png   EvertionBook-p62.png