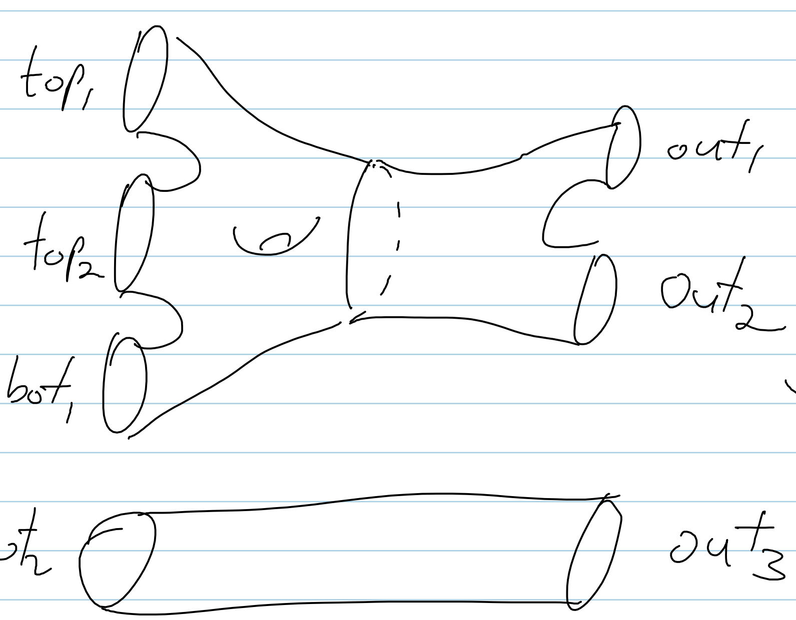
VCLaw - the math

June-17-13
11:19 AM

I wish I had my pensieve up the first time around I thought about these things.

Abstractly, VC is achieved by sticking the input disks into "generalized pants":



The "size" of a component is the number of bars it touches.

1. Dots slide to the minimal out disk on their component.

2. The genus of a component is

$$\frac{1}{2}\left[2+\frac{1}{2}(\text{size}) - \#\begin{pmatrix}\text{bndry}\\ \text{disks}\end{pmatrix}\right]$$

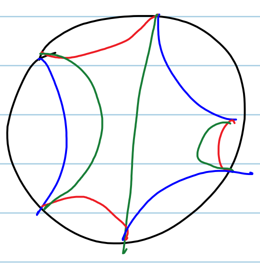
$\chi = 2-2g$

$g = \frac{2-\chi}{2}$

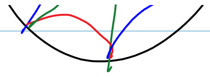
Example: saddle $\circ$ saddle: $\frac{1}{2}\left(2+\frac{1}{2}(4)-4\right) = 0$

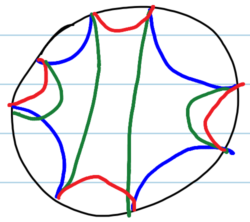
Example: $\text{identity}_{2n} \circ \begin{smallmatrix}\text{one}\\ \text{disk}\end{smallmatrix}$: $\frac{1}{2}\left(2+n-(2+n)\right)=0$

Example:

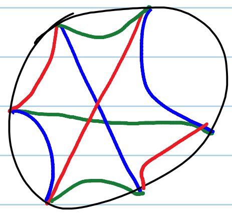


$\frac{1}{2}(2+3-1-2-2) = 0$

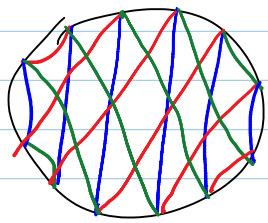




$\frac{1}{2}(2+4-1-3-2)=0$



$\frac{1}{2}(2+3-3\cdot 1)=1$



$\frac{1}{2}(2+6-1-2-1)=2$

3. genus ≥ 2 : composition is 0.

genus 1 : Any dot in input $\Rightarrow$ 0

No dots in input $\Rightarrow$ all outputs are dotted.

genus 0 : ≥ 2 dots in input $\Rightarrow$ 0

One dot in input $\Rightarrow$ all outputs are dotted.

No dots in input $\Rightarrow$ sum of all ways of having all outs but one dotted.

4. Therefore, "decor" may simply be the "no dots" output in each case; and after evaluations, squares must be removed.