Mina Aganagic on Knots and string dualities

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"Knot homology and the index"

$J_K(q) \longleftrightarrow$ The Jones polynomial, comes from $SU(2)$ CS on S^3 w/ Wilson Loop

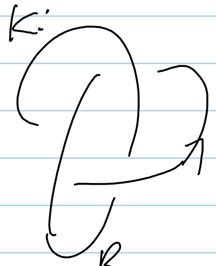
$$Z(M) = \int \mathcal{D}A \, e^{ik S_{cs}(M)}$$

no metric!

$$S_{cs} = \int_M A \wedge dA + \frac{2}{3} A \wedge A \wedge A \qquad k = \frac{1}{\hbar}$$

... topological invariants of M.

K:



R

$$\longrightarrow \operatorname{Tr}_R P e^{\int_K iA} =: \mathcal{O}_R(K) \qquad e^{\hbar} \sim$$

$$J_K(q) = \langle \mathcal{O}_R(K) \rangle_{cs} \qquad q = e^{2\pi i/k+2}$$

Why do it? Why is it important?

Dror: It isn't.

$\Rightarrow$ makes invariance manifest. [f.t. invariants are easier]

$\Rightarrow$ "The Jones poly is a quantum object"

$\Rightarrow$ "Chern-Simons theory provides the logic behind the zoo of invariants".

Weak reasons!

$\Rightarrow$ CS arises elsewhere, and connections are made.

Chern-Simons Theory is Soluble

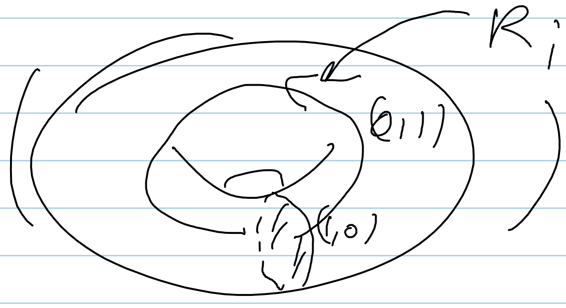
CS invariants of any 3-manifold [@ G, k]
are computable explicitly in terms of
S $\quad T$ $\quad B$ ("Reshetikhin-Turaev invariants")

path integral on a manifold M w/
$\partial M = B$ determines a state

$$Z(M) \in \mathcal{H}_B$$ "The Hilbert space of B"

In 3D TQFT, $B = T^2$

$\mathcal{H}_B = \mathcal{H}_{T^2}$ is spanned by

$Z\Big($ 

$\Big) =: |R_i\rangle$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$
$$S^2 = -1, \; (ST)^3 = S^2$$

$SL(2, \mathbb{Z})$ acts on $\mathcal{H}_{T^2}$
$\updownarrow$
diffeos of T^2

$S \longleftrightarrow$ invariants of Hopf-links

B: The braiding matrix.

.... Every 3-manifold is a surgery on a link....

Write an arbitrary link as a braid closure, use B .....

What CS does not explain:

One always gets polynomials w/ <u>integer</u> coefficients.

Khovanov 99':

$$K \to \mathcal{H}^{i,j}(K) \leftarrow \text{v.s. w/ 2 gradings}$$

s.t. $$J_K(q) = \sum_{i,j} (-1)^j q^i \dim\left(\mathcal{H}^{i,j}(K)\right)$$