Marcos Marino on Integrality properties of link invariants from topological-strings, Talk 2

June-27-13
8:58 AM

Coloured HOMFLY: Oriented links, $R_1 \ldots R_k$ $\mathcal{H}_{R_1 \ldots R_k}(L)$

Colored Kauffman: unoriented links, $R_1 \ldots R_k$, $g_{R_1 \ldots R_k}(L)$

Generating functions:

$$Z_{\mathcal{H}} = \sum_R \mathcal{H}_R(K)\, S_R(V)$$

Schur polys

[has some gauge theory explanation]

Free energy: $F_{\mathcal{H}}(V) = \log Z_{\mathcal{H}}(V)$

in some Schur-algebra sense.

"Reformulated Coloured Invariants $f_R(t,V)$":

$$F_{\mathcal{H}}(V) = \sum_{d=1}^{\infty} \sum_R \frac{1}{d} f_R(t^d, V^d)\, S_R(V^d)$$

$$\mathrm{Exp} = \exp\left(\sum_{d=1}^{\infty} \frac{1}{d} \Psi_d\right) \qquad \Psi_d: \text{ Adams ops}$$

"Plethystic exponential"

$$Z_{\mathcal{H}}(V) = \mathrm{Exp}\left(\sum_R f_R(t,V)\, S_R(V)\right)$$

It can that the f_R are completely determined by the $\mathcal{H}_R$:

$$f_{\square} = \mathcal{H}_{\square}$$

$$f_{\square\square} = \mathcal{H}_{\square\square} - \frac{1}{2}\mathcal{H}_{\square}^2 - \frac{1}{2}\mathcal{H}_{\square}(t^2, v^2) \qquad \text{etc.}$$

Define a matrix

Define a (invertible) matrix: μ partition, C_μ conjugacy class of $S_{|\mu|}$

$|\mu| = \sum \mu_i$

$\chi_R(C_\mu)$ character in the rep R

$$M_{RS} = \begin{cases} 0 & \text{if } \ell(R) \neq \ell(S) \\ \displaystyle\sum_\mu \frac{1}{z_\mu} \chi_R(C_\mu)\chi_S(C_\mu) \frac{\prod_{i \geq 1} (t^{\mu_i} - t^{-\mu_i})}{t - t^{-1}} \end{cases}$$

$$z_\mu = \frac{|\mu|!}{|C_\mu|}$$

$\ell(R) = \#$ boxes in R

$$\hat{f}_R(t, v) = \sum_S M^{-1}_{RS} f_S(t, v)$$

LMOV conjecture (Labastida, Marino, Ooguri, Vafa)
(proven by Lin-Peng)

$$\hat{f}_R(t, v) \in z^{-1}\mathbb{Z}[z^2, v^{\pm 1}] \qquad z \stackrel{?}{=} t^r - t^{-r}$$

For links,

$$Z_{\mathcal{H}}(v_1 \ldots v_L) = \sum_{R_1 \ldots R_L} \mathcal{H}_{R_1 \ldots R_L}(\mathcal{L}) S_{R_1}(v_1) \ldots S_{R_L}(v_L)$$

each v_ℓ is an infinite set of variables