Pensieve header: Testing ConciseFastKh.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-06\\Archive"];
<< KnotTheory`
<< "ConciseFastKh-130626-0938.m"
<< "../ConciseFastKh-Utilities.m"
```

Loading KnotTheory` version of February 5, 2013, 3:48:46.4762.
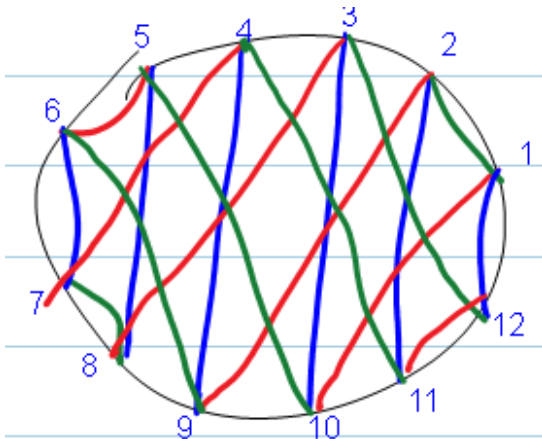Read more at http://katlas.org/wiki/KnotTheory.

```
c1 = Cob[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]], dot[1]]
```

Cob[S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]], dot[1]]

```
{ECP[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]]],
 ECR[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]]]}
```

$\{\{1 \to 1, 2 \to 1, 3 \to 1, 4 \to 1\}, \{1\}\}$

```
{β = S[P[1, 2], P[3, 4]], τ = S[P[2, 3], P[1, 4]]}
```

{S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]]}

```
{ECP[β, τ], ECR[β, τ]}
```

$\{\{1 \to 1, 2 \to 1, 3 \to 1, 4 \to 1\}, \{1\}\}$


```
{β = S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 τ = S[P[1, 10], P[2, 9], P[3, 8], P[4, 7], P[5, 6], P[11, 12]],
 μ = S[P[1, 12], P[2, 11], P[3, 10], P[4, 9], P[5, 8], P[6, 7]]}
```

{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 S[P[1, 10], P[2, 9], P[3, 8], P[4, 7], P[5, 6], P[11, 12]],
 S[P[1, 12], P[2, 11], P[3, 10], P[4, 9], P[5, 8], P[6, 7]]}

```
{ECP[β, τ], ECP[β, μ], ECP[μ, τ], ECP[β, τ, μ]}
```

$\{\{1 \to 1, 2 \to 1, 3 \to 3, 4 \to 3, 5 \to 1, 6 \to 1, 7 \to 3, 8 \to 3, 9 \to 1, 10 \to 1, 11 \to 3, 12 \to 3\},$
$\{1 \to 1, 2 \to 1, 3 \to 1, 4 \to 1, 5 \to 1, 6 \to 1, 7 \to 1, 8 \to 1, 9 \to 1, 10 \to 1, 11 \to 1, 12 \to 1\},$
$\{1 \to 1, 2 \to 1, 3 \to 1, 4 \to 1, 5 \to 1, 6 \to 1, 7 \to 1, 8 \to 1, 9 \to 1, 10 \to 1, 11 \to 1, 12 \to 1\},$
$\{1 \to 1, 2 \to 1, 3 \to 1, 4 \to 1, 5 \to 1, 6 \to 1, 7 \to 1, 8 \to 1, 9 \to 1, 10 \to 1, 11 \to 1, 12 \to 1\}\}$

{β /@ Range[4], τ /@ Range[4]}

{{2, 1, 12, 11}, {10, 9, 8, 7}}

VCLaw[β, μ, τ]

{{dot[1] → dot[1]}, {dot[1] → dot[1]}, 0}

{β, m[4, 11][β], m[1, 5][β]}

{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 S[P[1, 2], P[3, 12], P[5, 10], P[6, 9], P[7, 8]],
 S[P[2, 10], P[3, 12], P[4, 11], P[6, 9], P[7, 8]]}

{β, m[4, 11][Q[2] β], m[1, 5][Q[3] β]}

{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 {Q[3] S[P[1, 2], P[3, 12], P[5, 10], P[6, 9], P[7, 8]],
  Q[1] S[P[1, 2], P[3, 12], P[5, 10], P[6, 9], P[7, 8]]},
 {Q[3] S[P[2, 10], P[3, 12], P[4, 11], P[6, 9], P[7, 8]]}}

Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[1]] // m[2, 3]

{{dot[1]}}

Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[2]] // m[2, 3]

{{dot[1]}}

Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[3]] // m[2, 3]

{{dot[1]}}

Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[4]] // m[2, 3]

{{dot[1]}}

Vect[Q[1] S[P[i, j], P[k, l]]]⊗Vect[Q[2] S[P[i, l], P[j, k]]]

Vect[Q[1] S[P[i, j], P[k, l]]]⊗Vect[Q[2] S[P[i, l], P[j, k]]]

Kom[{{S[]}}, {}] Cob[S[P[9, 10], P[11, 12]], Q[1] S[P[9, 12], P[10, 11]], 1]

Kom[{{S[P[9, 10], P[11, 12]]}, {Q[1] S[P[9, 12], P[10, 11]]}}, {{{-1}}}]

KhPoly[Knot[3, 1]]

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\frac{1}{q^3} + \frac{1}{q} + \frac{1}{q^9 t^3} + \frac{1}{q^5 t^2}$$

Kh[Knot[3, 1]][q, t]

KnotTheory::loading : Loading precomputed data in Kh4Knots`.

$$\frac{1}{q^3} + \frac{1}{q} + \frac{1}{q^9 t^3} + \frac{1}{q^5 t^2}$$

KhPoly[Knot[6, 2]]

$$\frac{1}{q^3} + \frac{2}{q} + \frac{1}{q^{11} t^4} + \frac{1}{q^9 t^3} + \frac{1}{q^7 t^3} + \frac{1}{q^7 t^2} + \frac{1}{q^5 t^2} + \frac{1}{q^5 t} + \frac{1}{q^3 t} + \frac{t}{q} + q^3 t^2$$

Kh[Knot[6, 2]][q, t]

$$\frac{1}{q^3} + \frac{2}{q} + \frac{1}{q^{11} t^4} + \frac{1}{q^9 t^3} + \frac{1}{q^7 t^3} + \frac{1}{q^7 t^2} + \frac{1}{q^5 t^2} + \frac{1}{q^5 t} + \frac{1}{q^3 t} + \frac{t}{q} + q^3 t^2$$

KhPoly[Knot[8, 17]] // Timing

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\Big\{ 3.198021, \frac{4}{q} + 4\, q + \frac{1}{q^9 t^4} + \frac{2}{q^7 t^3} + \frac{1}{q^5 t^3} + \frac{3}{q^5 t^2} +$$

$$\frac{2}{q^3 t^2} + \frac{3}{q^3 t} + \frac{3}{q t} + 3\, q\, t + 3\, q^3\, t + 2\, q^3\, t^2 + 3\, q^5\, t^2 + q^5\, t^3 + 2\, q^7\, t^3 + q^9\, t^4 \Big\}$$

Kh[Knot[8, 17]][q, t]

$$\frac{4}{q} + 4\, q + \frac{1}{q^9 t^4} + \frac{2}{q^7 t^3} + \frac{1}{q^5 t^3} + \frac{3}{q^5 t^2} + \frac{2}{q^3 t^2} +$$

$$\frac{3}{q^3 t} + \frac{3}{q t} + 3\, q\, t + 3\, q^3\, t + 2\, q^3\, t^2 + 3\, q^5\, t^2 + q^5\, t^3 + 2\, q^7\, t^3 + q^9\, t^4$$

KhPoly[Knot[10, 165]] // Timing

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\Big\{ 5.850038, 2\, q + q^3 + 3\, q^3\, t + q^5\, t + 3\, q^5\, t^2 + 4\, q^7\, t^2 + 4\, q^7\, t^3 + 3\, q^9\, t^3 +$$

$$4\, q^9\, t^4 + 3\, q^{11}\, t^4 + 2\, q^{11}\, t^5 + 4\, q^{13}\, t^5 + 2\, q^{13}\, t^6 + 2\, q^{15}\, t^6 + q^{15}\, t^7 + 2\, q^{17}\, t^7 + q^{19}\, t^8 \Big\}$$

Kh[Knot[10, 165]][q, t]

KnotTheory::loading : Loading precomputed data in Kh4Knots`.

$$2\, q + q^3 + 3\, q^3\, t + q^5\, t + 3\, q^5\, t^2 + 3\, q^7\, t^2 + 3\, q^7\, t^3 + 3\, q^9\, t^3 + 4\, q^9\, t^4 +$$

$$3\, q^{11}\, t^4 + 2\, q^{11}\, t^5 + 4\, q^{13}\, t^5 + 2\, q^{13}\, t^6 + 2\, q^{15}\, t^6 + q^{15}\, t^7 + 2\, q^{17}\, t^7 + q^{19}\, t^8$$

KhPoly[TorusKnot[6, 5]] // Timing

$$\Big\{ 176.078329,$$

$$q^{19} + q^{21} + q^{23}\, t^2 + q^{27}\, t^3 + q^{25}\, t^4 + q^{27}\, t^4 + q^{29}\, t^5 + q^{31}\, t^5 + q^{27}\, t^6 + q^{29}\, t^6 + q^{31}\, t^7 + q^{33}\, t^7 + q^{29}\, t^8 +$$

$$2\, q^{31}\, t^8 + q^{33}\, t^9 + 2\, q^{35}\, t^9 + q^{33}\, t^{10} + 2\, q^{37}\, t^{11} + q^{35}\, t^{12} + q^{37}\, t^{12} + q^{41}\, t^{12} + q^{39}\, t^{13} + q^{41}\, t^{13} \Big\}$$

KhPoly[TorusKnot[9, 5]] // Timing

$$\Big\{ 837.506969, q^{31} + q^{33} + q^{35}\, t^2 + q^{39}\, t^3 + q^{37}\, t^4 + q^{39}\, t^4 + q^{41}\, t^5 + q^{43}\, t^5 + q^{39}\, t^6 + q^{41}\, t^6 + q^{43}\, t^7 +$$

$$q^{45}\, t^7 + q^{41}\, t^8 + 2\, q^{43}\, t^8 + q^{45}\, t^9 + 2\, q^{47}\, t^9 + 2\, q^{45}\, t^{10} + 3\, q^{49}\, t^{11} + 2\, q^{47}\, t^{12} + 2\, q^{49}\, t^{12} + q^{53}\, t^{12} +$$

$$3\, q^{51}\, t^{13} + 2\, q^{53}\, t^{13} + q^{49}\, t^{14} + 2\, q^{51}\, t^{14} + q^{55}\, t^{14} + 2\, q^{53}\, t^{15} + 3\, q^{55}\, t^{15} + 2\, q^{53}\, t^{16} + q^{57}\, t^{16} +$$

$$q^{59}\, t^{16} + 3\, q^{57}\, t^{17} + q^{55}\, t^{18} + q^{57}\, t^{18} + q^{61}\, t^{18} + 2\, q^{59}\, t^{19} + q^{61}\, t^{19} + q^{59}\, t^{20} + q^{63}\, t^{20} + q^{63}\, t^{21} \Big\}$$

KhPoly[TorusKnot[7, 6]] // Timing

$\{11\,112.965237, q^{29} + q^{31} + q^{33}\,t^2 + q^{37}\,t^3 + q^{35}\,t^4 + q^{37}\,t^4 + q^{39}\,t^5 + q^{41}\,t^5 + q^{37}\,t^6 + q^{39}\,t^6 +$
$\quad q^{41}\,t^7 + q^{43}\,t^7 + q^{39}\,t^8 + 2\,q^{41}\,t^8 + q^{43}\,t^9 + 2\,q^{45}\,t^9 + q^{41}\,t^{10} + 2\,q^{43}\,t^{10} + q^{45}\,t^{11} + 3\,q^{47}\,t^{11} +$
$\quad 2\,q^{45}\,t^{12} + q^{47}\,t^{12} + q^{51}\,t^{12} + 3\,q^{49}\,t^{13} + q^{51}\,t^{13} + q^{47}\,t^{14} + q^{49}\,t^{14} + q^{53}\,t^{14} + 2\,q^{51}\,t^{15} +$
$\quad 2\,q^{53}\,t^{15} + q^{49}\,t^{16} + q^{51}\,t^{16} + q^{55}\,t^{16} + q^{57}\,t^{16} + q^{53}\,t^{17} + q^{55}\,t^{17} + q^{53}\,t^{18} + q^{57}\,t^{19}\}$

((# → KhPoly[#] == Kh[#][q, t] & /@ AllKnots[8])) // Timing

KnotTheory::loading : Loading precomputed data in Kh4Knots`.

$\{14.320892, \{\text{Knot}[8, 1] \to \text{True}, \text{Knot}[8, 2] \to \text{True}, \text{Knot}[8, 3] \to \text{True},$
$\quad \text{Knot}[8, 4] \to \text{True}, \text{Knot}[8, 5] \to \text{True}, \text{Knot}[8, 6] \to \text{True}, \text{Knot}[8, 7] \to \text{True},$
$\quad \text{Knot}[8, 8] \to \text{True}, \text{Knot}[8, 9] \to \text{True}, \text{Knot}[8, 10] \to \text{True}, \text{Knot}[8, 11] \to \text{True},$
$\quad \text{Knot}[8, 12] \to \text{True}, \text{Knot}[8, 13] \to \text{True}, \text{Knot}[8, 14] \to \text{True},$
$\quad \text{Knot}[8, 15] \to \text{True}, \text{Knot}[8, 16] \to \text{True}, \text{Knot}[8, 17] \to \text{True}, \text{Knot}[8, 18] \to \text{True},$
$\quad \text{Knot}[8, 19] \to \text{True}, \text{Knot}[8, 20] \to \text{True}, \text{Knot}[8, 21] \to \text{True}\}\}$

KhComplex[Knot[8, 21]] // Show

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\begin{pmatrix} \frac{s[]}{q^{15}} & \frac{s[]}{q^{11}} & \frac{s[]}{q^{11}} & \frac{s[]}{q^9} & \frac{s[]}{q^5} & \frac{s[]}{q^3} & \frac{s[]}{q} \\[4pt] & \frac{s[]}{q^{13}} & \frac{s[]}{q^9} & \frac{s[]}{q^7} & \frac{s[]}{q^7} & \frac{s[]}{q^5} & \frac{s[]}{q^3} \\[4pt] & & & \frac{s[]}{q^9} & \frac{s[]}{q^5} & & \frac{s[]}{q} \\[4pt] \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} & 0 \quad 0 \quad 0 \end{pmatrix}$$

(Plus @@ (KhPoly[#] == Kh[#][q, t] & /@ AllKnots[{3, 10}])) // Timing

KnotTheory::loading : Loading precomputed data in PD4Knots`.

KnotTheory::loading : Loading precomputed data in Kh4Knots`.

$\{1414.445467, 249\ \text{True}\}$

(Plus @@ (KhPoly[#] == Kh[#][q, t] & /@ AllKnots[11])) // Timing

KnotTheory::loading : Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory::credits :

   The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

KnotTheory::loading : Loading precomputed data in Kh4Knots11`.