

Pensieve header: Testing ConciseFastKh.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-06\\Archive"];
<< KnotTheory`
<< "ConciseFastKh-130623-1304.m"
<< "../ConciseFastKh-Utilities.m"
```

Loading KnotTheory` version of February 5, 2013, 3:48:46.4762.  
Read more at <http://katlas.org/wiki/KnotTheory>.

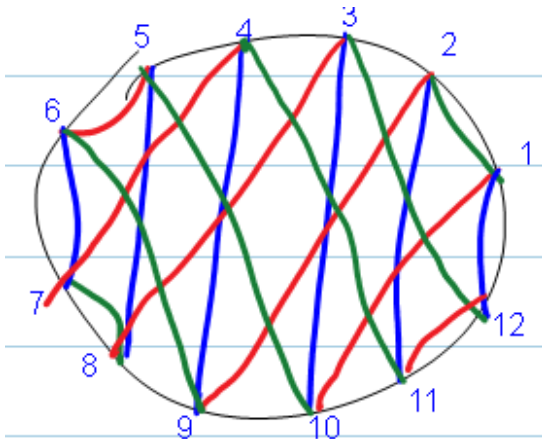
```
c1 = Cob[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]], dot[1]]
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]], dot[1]]
```

```
{ECP[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]]],
 ECR[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]]]}
{{1 → 1, 2 → 1, 3 → 1, 4 → 1}, {1}}
```

```
{β = S[P[1, 2], P[3, 4]], τ = S[P[2, 3], P[1, 4]]}
{S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]]}
```

```
{ECP[β, τ], ECR[β, τ]}
{{1 → 1, 2 → 1, 3 → 1, 4 → 1}, {1}}
```



```
{β = S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 τ = S[P[1, 10], P[2, 9], P[3, 8], P[4, 7], P[5, 6], P[11, 12]],
 μ = S[P[1, 12], P[2, 11], P[3, 10], P[4, 9], P[5, 8], P[6, 7]]}
{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 S[P[1, 10], P[2, 9], P[3, 8], P[4, 7], P[5, 6], P[11, 12]],
 S[P[1, 12], P[2, 11], P[3, 10], P[4, 9], P[5, 8], P[6, 7]]}
```

```
{ECP[β, τ], ECP[β, μ], ECP[μ, τ], ECP[β, τ, μ]}
```

```
{{1 → 1, 2 → 1, 3 → 3, 4 → 3, 5 → 1, 6 → 1, 7 → 3, 8 → 3, 9 → 1, 10 → 1, 11 → 3, 12 → 3},
 {1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1, 9 → 1, 10 → 1, 11 → 1, 12 → 1},
 {1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1, 9 → 1, 10 → 1, 11 → 1, 12 → 1},
 {1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1, 9 → 1, 10 → 1, 11 → 1, 12 → 1}}
```

```
{β /@ Range[4], τ /@ Range[4]}
```

```
{{2, 1, 12, 11}, {10, 9, 8, 7}}
```

```
VCLaw[β, μ, τ]
```

```
{{dot[1] → dot[1]}, {dot[1] → dot[1]}, 0}
```

```
{β, m[4, 11][β], m[1, 5][β]}
```

```
{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 S[P[1, 2], P[3, 12], P[5, 10], P[6, 9], P[7, 8]],
 S[P[2, 10], P[3, 12], P[4, 11], P[6, 9], P[7, 8]]}
```

```
{β, m[4, 11][Q[2] β], m[1, 5][Q[3] β]}
```

```
{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 {Q[3] S[P[1, 2], P[3, 12], P[5, 10], P[6, 9], P[7, 8]],
 Q[1] S[P[1, 2], P[3, 12], P[5, 10], P[6, 9], P[7, 8]]},
 {Q[3] S[P[2, 10], P[3, 12], P[4, 11], P[6, 9], P[7, 8]]}}
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[1]] // m[2, 3]
```

```
{{dot[1]}}
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[2]] // m[2, 3]
```

```
{{dot[1]}}
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[3]] // m[2, 3]
```

```
{{dot[1]}}
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[4]] // m[2, 3]
```

```
{{dot[1]}}
```

```
Vect[Q[1] S[P[i, j], P[k, l]]] ⊗ Vect[Q[2] S[P[i, l], P[j, k]]]
```

```
Vect[Q[1] S[P[i, j], P[k, l]]] ⊗ Vect[Q[2] S[P[i, l], P[j, k]]]
```

```
Kom[{{S[]}}, {}] Cob[S[P[9, 10], P[11, 12]], Q[1] S[P[9, 12], P[10, 11]], 1]
```

```
Kom[{{S[P[9, 10], P[11, 12]]}, {Q[1] S[P[9, 12], P[10, 11]]}}, {{{-1}}}]
```

```
CFKh[Knot[3, 1]]
```

```
KnotTheory::loading: Loading precomputed data in PD4Knots`.
```

$$\frac{1}{q^3} + \frac{1}{q} + \frac{1}{q^9 t^3} + \frac{1}{q^5 t^2}$$

```
Kh[Knot[3, 1]][q, t]
```

```
KnotTheory::loading: Loading precomputed data in Kh4Knots`.
```

$$\frac{1}{q^3} + \frac{1}{q} + \frac{1}{q^9 t^3} + \frac{1}{q^5 t^2}$$

**CFKh[Knot[6, 2]]**

$$\frac{1}{q^3} + \frac{2}{q} + \frac{1}{q^{11}t^4} + \frac{1}{q^9t^3} + \frac{1}{q^7t^3} + \frac{1}{q^7t^2} + \frac{1}{q^5t^2} + \frac{1}{q^5t} + \frac{1}{q^3t} + \frac{t}{q} + q^3t^2$$

**Kh[Knot[6, 2]][q, t]**

$$\frac{1}{q^3} + \frac{2}{q} + \frac{1}{q^{11}t^4} + \frac{1}{q^9t^3} + \frac{1}{q^7t^3} + \frac{1}{q^7t^2} + \frac{1}{q^5t^2} + \frac{1}{q^5t} + \frac{1}{q^3t} + \frac{t}{q} + q^3t^2$$

**CFKh[Knot[8, 17]] // Timing**

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\left\{ 3.198021, \frac{4}{q} + 4q + \frac{1}{q^9t^4} + \frac{2}{q^7t^3} + \frac{1}{q^5t^3} + \frac{3}{q^5t^2} + \frac{2}{q^3t^2} + \frac{3}{q^3t} + \frac{3}{qt} + 3qt + 3q^3t + 2q^3t^2 + 3q^5t^2 + q^5t^3 + 2q^7t^3 + q^9t^4 \right\}$$

**Kh[Knot[8, 17]][q, t]**

$$\frac{4}{q} + 4q + \frac{1}{q^9t^4} + \frac{2}{q^7t^3} + \frac{1}{q^5t^3} + \frac{3}{q^5t^2} + \frac{2}{q^3t^2} + \frac{3}{q^3t} + \frac{3}{qt} + 3qt + 3q^3t + 2q^3t^2 + 3q^5t^2 + q^5t^3 + 2q^7t^3 + q^9t^4$$

**CFKh[Knot[10, 165]] // Timing**

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\left\{ 7.035645, 2q + q^3 + 3q^3t + q^5t + 3q^5t^2 + 3q^7t^2 + 3q^7t^3 + 3q^9t^3 + 4q^9t^4 + 3q^9t^4 + 3q^{11}t^4 + 2q^{11}t^5 + 4q^{13}t^5 + 2q^{13}t^6 + 2q^{15}t^6 + q^{15}t^7 + 2q^{17}t^7 + q^{19}t^8 \right\}$$

**Kh[Knot[10, 165]][q, t]**

KnotTheory::loading : Loading precomputed data in Kh4Knots`.

$$2q + q^3 + 3q^3t + q^5t + 3q^5t^2 + 3q^7t^2 + 3q^7t^3 + 3q^9t^3 + 4q^9t^4 + 3q^{11}t^4 + 2q^{11}t^5 + 4q^{13}t^5 + 2q^{13}t^6 + 2q^{15}t^6 + q^{15}t^7 + 2q^{17}t^7 + q^{19}t^8$$

**CFKh[TorusKnot[6, 5]] // Timing**

$$\left\{ 249.305198, q^{19} + q^{21} + q^{23}t^2 + q^{27}t^3 + q^{25}t^4 + q^{27}t^4 + q^{29}t^5 + q^{31}t^5 + q^{27}t^6 + q^{29}t^6 + q^{31}t^7 + q^{33}t^7 + q^{29}t^8 + 2q^{31}t^8 + q^{33}t^9 + 2q^{35}t^9 + q^{33}t^{10} + 2q^{37}t^{11} + q^{35}t^{12} + q^{37}t^{12} + q^{41}t^{12} + q^{39}t^{13} + q^{41}t^{13} \right\}$$

**CFKh[TorusKnot[9, 5]] // Timing**

$$\left\{ 837.506969, q^{31} + q^{33} + q^{35}t^2 + q^{39}t^3 + q^{37}t^4 + q^{39}t^4 + q^{41}t^5 + q^{43}t^5 + q^{39}t^6 + q^{41}t^6 + q^{43}t^7 + q^{45}t^7 + q^{41}t^8 + 2q^{43}t^8 + q^{45}t^9 + 2q^{47}t^9 + 2q^{45}t^{10} + 3q^{49}t^{11} + 2q^{47}t^{12} + 2q^{49}t^{12} + q^{53}t^{12} + 3q^{51}t^{13} + 2q^{53}t^{13} + q^{49}t^{14} + 2q^{51}t^{14} + q^{55}t^{14} + 2q^{53}t^{15} + 3q^{55}t^{15} + 2q^{53}t^{16} + q^{57}t^{16} + q^{59}t^{16} + 3q^{57}t^{17} + q^{55}t^{18} + q^{57}t^{18} + q^{61}t^{18} + 2q^{59}t^{19} + q^{61}t^{19} + q^{59}t^{20} + q^{63}t^{20} + q^{63}t^{21} \right\}$$

```
CFKh[TorusKnot[7, 6]] // Timing
```

```
{11 112.965237, q29 + q31 + q33 t2 + q37 t3 + q35 t4 + q37 t4 + q39 t5 + q41 t5 + q37 t6 + q39 t6 +
  q41 t7 + q43 t7 + q39 t8 + 2 q41 t8 + q43 t9 + 2 q45 t9 + q41 t10 + 2 q43 t10 + q45 t11 + 3 q47 t11 +
  2 q45 t12 + q47 t12 + q51 t12 + 3 q49 t13 + q51 t13 + q47 t14 + q49 t14 + q53 t14 + 2 q51 t15 +
  2 q53 t15 + q49 t16 + q51 t16 + q55 t16 + q57 t16 + q53 t17 + q55 t17 + q53 t18 + q57 t19}
```

```
{DateString[], Plus @@ (CFKh[#] == Kh[#][q, t] & /@ AllKnots[{3, 10}])} // Timing
```

```
KnofTheory::loading: Loading precomputed data in PD4Knots`.
```

```
KnofTheory::loading: Loading precomputed data in Kh4Knots`.
```

```
{860.891518, {Sat 29 Jun 2013 14:32:22, 249 True}}
```

```
KhComplex[L_] := Module[
```

```
{pd = PD[L], kom = Kom[], inside = {}, pos},
```

```
While[Length[pd] > 0,
```

```
pos = Last[Ordering[(Length[(List @@ #) ∩ inside]) & /@ pd]];

```

```
kom = kom // Cob[pd[[pos]]];

```

```
(kom = Contract[kom // m[#, -#]]) & /@ ((List @@ pd[[pos]]) ∩ inside);

```

```
inside = inside ∪ (List @@ pd[[pos]]); pd = Drop[pd, {pos}];

```

```
kom];
```

```
KhPoly[L_] := Expand[t-Length@Select[PD@L, NegativeQ]+Range[0, Crossings[L]].
```

```
(List @@ Plus @@@ First @ KhComplex[L]) /. S[] → 1]
```