

Pensieve header: Testing ConciseFastKh.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-06"];
```

```
<< ConciseFastKh.m
```

Loading KnotTheory` version of February 5, 2013, 3:48:46.4762.
Read more at <http://katlas.org/wiki/KnotTheory>.

```
c1 = Cob[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]], dot[1]]
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]], dot[1]]
```

```
{ECP[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]]],
```

```
ECR[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]]]}
```

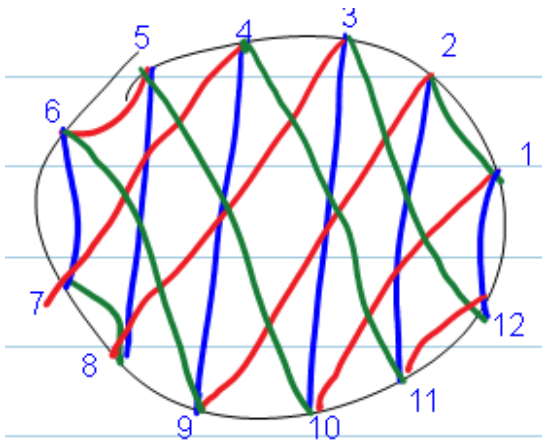
```
{{1 → 1, 2 → 1, 3 → 1, 4 → 1}, {1}}
```

```
{β = S[P[1, 2], P[3, 4]], τ = S[P[2, 3], P[1, 4]]}
```

```
{S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]]}
```

```
{ECP[β, τ], ECR[β, τ]}
```

```
{{1 → 1, 2 → 1, 3 → 1, 4 → 1}, {1}}
```



```
{β = S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
```

```
τ = S[P[1, 10], P[2, 9], P[3, 8], P[4, 7], P[5, 6], P[11, 12]],
```

```
μ = S[P[1, 12], P[2, 11], P[3, 10], P[4, 9], P[5, 8], P[6, 7]]}
```

```
{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
```

```
S[P[1, 10], P[2, 9], P[3, 8], P[4, 7], P[5, 6], P[11, 12]],
```

```
S[P[1, 12], P[2, 11], P[3, 10], P[4, 9], P[5, 8], P[6, 7]]}
```

```
{ECP[β, τ], ECP[β, μ], ECP[μ, τ], ECP[β, τ, μ]}
```

```
{{1 → 1, 2 → 1, 3 → 3, 4 → 3, 5 → 1, 6 → 1, 7 → 3, 8 → 3, 9 → 1, 10 → 1, 11 → 3, 12 → 3},
```

```
{1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1, 9 → 1, 10 → 1, 11 → 1, 12 → 1},
```

```
{1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1, 9 → 1, 10 → 1, 11 → 1, 12 → 1},
```

```
{1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1, 9 → 1, 10 → 1, 11 → 1, 12 → 1}}
```

```
{β /@ Range[4], τ /@ Range[4]}
```

```
{{2, 1, 12, 11}, {10, 9, 8, 7}}
```

VCLaw $[\beta, \mu, \tau]$

$\{\{\text{dot}[1] \rightarrow \text{dot}[1]\}, \{\text{dot}[1] \rightarrow \text{dot}[1]\}, 0\}$

β , **m** $[4, 11][\beta]$, **m** $[1, 5][\beta]$

$\{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],$
 $S[P[1, 2], P[3, 12], P[5, 10], P[6, 9], P[7, 8]],$
 $S[P[2, 10], P[3, 12], P[4, 11], P[6, 9], P[7, 8]]\}$

β , **m** $[4, 11][Q[2] \beta]$, **m** $[1, 5][Q[3] \beta]$

$\{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],$
 $Q[3] S[P[1, 2], P[3, 12], P[5, 10], P[6, 9], P[7, 8]],$
 $Q[1] S[P[1, 2], P[3, 12], P[5, 10], P[6, 9], P[7, 8]],$
 $Q[3] S[P[2, 10], P[3, 12], P[4, 11], P[6, 9], P[7, 8]]\}$

Cob $[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], \text{dot}[1]] // m[2, 3]$

$\{\{\text{dot}[1]\}\}$

Cob $[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], \text{dot}[2]] // m[2, 3]$

$\{\{\text{dot}[1]\}\}$

Cob $[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], \text{dot}[3]] // m[2, 3]$

$\{\{\text{dot}[1]\}\}$

Cob $[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], \text{dot}[4]] // m[2, 3]$

$\{\{\text{dot}[1]\}\}$

Vect $[Q[1] S[P[i, j], P[k, l]]] \otimes \text{Vect}[Q[2] S[P[i, l], P[j, k]]]$

$\text{Vect}[Q[1] S[P[i, j], P[k, l]]] \otimes \text{Vect}[Q[2] S[P[i, l], P[j, k]]]$

Kom $\{\{S\}, \{\}\} \text{Cob}[S[P[9, 10], P[11, 12]], Q[1] S[P[9, 12], P[10, 11]], 1]$

$\text{Kom}\{\{S[P[9, 10], P[11, 12]], \{Q[1] S[P[9, 12], P[10, 11]]\}\}, \{\{-1\}\}$

CFKh $[Knot[3, 1]]$

KnotTheory:loading: Loading precomputed data in PD4Knots`.

$$\frac{1}{q^3} + \frac{1}{q} + \frac{1}{q^9 t^3} + \frac{1}{q^5 t^2}$$

Kh $[Knot[3, 1]][q, t]$

KnotTheory:loading: Loading precomputed data in Kh4Knots`.

$$\frac{1}{q^3} + \frac{1}{q} + \frac{1}{q^9 t^3} + \frac{1}{q^5 t^2}$$

CFKh $[Knot[6, 2]]$

$$\frac{1}{q^3} + \frac{2}{q} + \frac{1}{q^{11} t^4} + \frac{1}{q^9 t^3} + \frac{1}{q^7 t^3} + \frac{1}{q^7 t^2} + \frac{1}{q^5 t^2} + \frac{1}{q^5 t} + \frac{1}{q^3 t} + \frac{t}{q} + q^3 t^2$$

Kh[Knot[6, 2]] [q, t]

$$\frac{1}{q^3} + \frac{2}{q} + \frac{1}{q^{11} t^4} + \frac{1}{q^9 t^3} + \frac{1}{q^7 t^3} + \frac{1}{q^7 t^2} + \frac{1}{q^5 t^2} + \frac{1}{q^5 t} + \frac{1}{q^3 t} + \frac{t}{q} + q^3 t^2$$

CFKh[Knot[8, 17]] // Timing

$$\left\{ 2.043613, \frac{4}{q} + 4q + \frac{1}{q^9 t^4} + \frac{2}{q^7 t^3} + \frac{1}{q^5 t^3} + \frac{3}{q^5 t^2} + \frac{2}{q^3 t^2} + \frac{3}{q^3 t} + \frac{3}{qt} + 3qt + 3q^3 t + 2q^3 t^2 + 3q^5 t^2 + q^5 t^3 + 2q^7 t^3 + q^9 t^4 \right\}$$

Kh[Knot[8, 17]] [q, t]

$$\frac{4}{q} + 4q + \frac{1}{q^9 t^4} + \frac{2}{q^7 t^3} + \frac{1}{q^5 t^3} + \frac{3}{q^5 t^2} + \frac{2}{q^3 t^2} + \frac{3}{q^3 t} + \frac{3}{qt} + 3qt + 3q^3 t + 2q^3 t^2 + 3q^5 t^2 + q^5 t^3 + 2q^7 t^3 + q^9 t^4$$

CFKh[Knot[10, 165]] // Timing

$$\left\{ 5.444435, 2q + q^3 + 3q^3 t + q^5 t + 3q^5 t^2 + 3q^7 t^2 + 3q^7 t^3 + 3q^9 t^3 + 4q^9 t^4 + 3q^{11} t^4 + 2q^{11} t^5 + 4q^{13} t^5 + 2q^{13} t^6 + 2q^{15} t^6 + q^{15} t^7 + 2q^{17} t^7 + q^{19} t^8 \right\}$$

Crossings[TorusKnot[6, 5]]

24

CFKh[TorusKnot[6, 5]] // Timing

$$\left\{ 223.112630, q^{19} + q^{21} + q^{23} t^2 + q^{27} t^3 + q^{25} t^4 + q^{27} t^4 + q^{29} t^5 + q^{31} t^5 + q^{27} t^6 + q^{29} t^6 + q^{31} t^7 + q^{33} t^7 + q^{29} t^8 + 2q^{31} t^8 + q^{33} t^9 + 2q^{35} t^9 + q^{33} t^{10} + 2q^{37} t^{11} + q^{35} t^{12} + q^{37} t^{12} + q^{41} t^{12} + q^{39} t^{13} + q^{41} t^{13} \right\}$$

CFKh[TorusKnot[9, 5]] // Timing

$$\left\{ 837.506969, q^{31} + q^{33} + q^{35} t^2 + q^{39} t^3 + q^{37} t^4 + q^{39} t^4 + q^{41} t^5 + q^{43} t^5 + q^{39} t^6 + q^{41} t^6 + q^{43} t^7 + q^{45} t^7 + q^{41} t^8 + 2q^{43} t^8 + q^{45} t^9 + 2q^{47} t^9 + 2q^{45} t^{10} + 3q^{49} t^{11} + 2q^{47} t^{12} + 2q^{49} t^{12} + q^{53} t^{12} + 3q^{51} t^{13} + 2q^{53} t^{13} + q^{49} t^{14} + 2q^{51} t^{14} + q^{55} t^{14} + 2q^{53} t^{15} + 3q^{55} t^{15} + 2q^{53} t^{16} + q^{57} t^{16} + q^{59} t^{16} + 3q^{57} t^{17} + q^{55} t^{18} + q^{57} t^{18} + q^{61} t^{18} + 2q^{59} t^{19} + q^{61} t^{19} + q^{59} t^{20} + q^{63} t^{20} + q^{63} t^{21} \right\}$$

CFKh[TorusKnot[7, 6]] // Timing

$$\left\{ 11112.965237, q^{29} + q^{31} + q^{33} t^2 + q^{37} t^3 + q^{35} t^4 + q^{37} t^4 + q^{39} t^5 + q^{41} t^5 + q^{37} t^6 + q^{39} t^6 + q^{41} t^7 + q^{43} t^7 + q^{39} t^8 + 2q^{41} t^8 + q^{43} t^9 + 2q^{45} t^9 + q^{41} t^{10} + 2q^{43} t^{10} + q^{45} t^{11} + 3q^{47} t^{11} + 2q^{45} t^{12} + q^{47} t^{12} + q^{51} t^{12} + 3q^{49} t^{13} + q^{51} t^{13} + q^{47} t^{14} + q^{49} t^{14} + q^{53} t^{14} + 2q^{51} t^{15} + 2q^{53} t^{15} + q^{49} t^{16} + q^{51} t^{16} + q^{55} t^{16} + q^{57} t^{16} + q^{53} t^{17} + q^{55} t^{17} + q^{53} t^{18} + q^{57} t^{19} \right\}$$