Roland van der Veen: A generating function for MOY graphs

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11:12 PM

$$\sum_k \binom{N}{k} z^k = (1+z)^N$$

Plan: 0. Motivation

1. MOY evaluation
   generating functions
   - unknot
   - $q=1$
   - general case
2. Applications.

First speaker to prop the boards up!

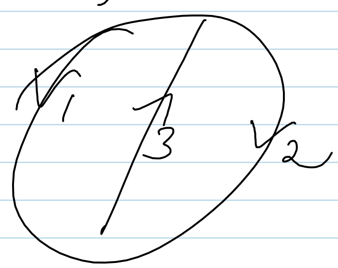
Motivation: Colored HOMFLY polynomial — recursions in a and color.
Simplify to $SL(N)$ quantum invariants of planar graphs

MOY graphs: An oriented planar trivalent graph Γ together with a flow $\gamma : E(\Gamma) \to \mathbb{N}$

Evaluation: Use

$$\Lambda^{a+b} V \longrightarrow \Lambda^a V \otimes \Lambda^b V$$

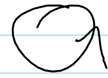
etc, take values in Laurent polys in q.



$$\langle \Gamma, \gamma \rangle$$

$$\langle \bigcirc, k \rangle = \binom{N}{k} \text{ or } \begin{bmatrix} N \\ k \end{bmatrix}_q$$

$$\sum \langle \bigcirc, k \rangle z^k = (\underbrace{1+z}_{})^N$$

cycles in Γ = 

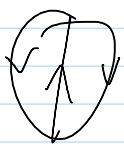
or $(1+z)^{\underline{N}}$ where $\underline{N} := \{ \frac{1-N}{2}, \ldots, \frac{N-1}{2} \}$

$$\begin{gathered}(1+z)^{\underline{N}} \\ \| \\ \prod_{k \in \underline{N}} (1+q^k z)\end{gathered}$$

There's also a version with a.

<u>Proposition</u> $\langle \Gamma, \gamma \rangle (q=1)$ is the number of states σ s.t. $|\sigma| = \gamma$

G: set of oriented cycles in Γ

 $\Rightarrow G = \{ [\,], D, \emptyset \}$

A state is a map $\sigma : G \to \text{subsets}(\underline{N})$

$|\sigma|$ defined using cardinalities. with some disjointness condition on the edges.

$$\sum_{\gamma} \langle \Gamma, \gamma \rangle_N (q=1)\, z^{\gamma} = \Big(\sum_{c \in G} z^c \Big)^N$$

where $z^{\gamma} := \prod_{e \in E(\Gamma)} z_e^{\gamma(e)}$

$=: F_{\Gamma, N}(z)$

<u>Example</u> $F_{x\phi z, 3}(x, y, z) =$

$(1+xy+yz)^3$

There is also a story about symmetric powers ......

MOY evaluations for general q:

$\langle \Gamma, \gamma \rangle = \sum q^{rot(C)} q^{\frac{1}{4}\sum_{v\in V(\Gamma)} wt(E,v}$