Julien Marche: Knot state asymptotics (1)

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① statement: M: closed 3-manifold.

WRT: $Z_k(M) = \int_{\mathcal{A}/\mathcal{G}} \mathcal{D}\alpha\, e^{ikCS(\alpha)}$ $\alpha \in \Omega^1(M, su(2))$

$$CS(\alpha) = \frac{1}{4\pi}\int_M Tr\left(\alpha \wedge d\alpha + \frac{2}{3}\alpha\wedge\alpha\wedge\alpha\right)$$

stationary phase:

critical points $\Leftrightarrow$ flat connections $d\alpha + \frac{1}{2}[\alpha\wedge]$

// gauge equiv $\Leftrightarrow$ $\mathcal{M}(M, su(2)) = Hom(\pi_1(M), SU(2))/SU(2)$

Conjecture: If

$$Z_k(M) \sim \sum_{\rho \in Irr\,\mathcal{M}} e^{i m_\rho \pi/4 + ikCS(\rho)} \sqrt{\tau(M,\rho)} + O\left(\frac{1}{\sqrt{k}}\right)$$

$\#(M,\rho)$: Reid. Torsion / Ray-Singer.

$C^*(\ )$ .... defined in the cellular manner as a determinant.

makes sense when ρ is irred. & isolated.

Hypothesis: $\forall \rho \in Irr$, we have $H^1(M, ad\rho) = 0$

(namely, all irreds are isolated)

Proved for many Seifert manifolds

Jeffrey, Rozansky, Hikami, Hanson

for mapping tory of finite order diffeos, (Andersen)

for arbitrary mapping tori (L. Charles)

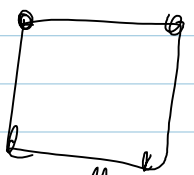
Thm (Marché, L. Charles)

Let $M_{p/q}$ be the Dehn filling of the complement of the figure-8 knot w/ slope p/q (some exclusions only due to irr & isolation conditions). Then conjecture holds.

The definition of WRT.

Strategy of proof.

Find a Geometric model for $V_k(\Sigma)$ ← TQFT Hilbert space of Σ?



"The pillowcase"