Pensieve header: Finding a trees-and-wheels associator using μ-calculus technology.

Finding a Trees-and-Wheels Associator

Preliminaries, Loading V

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-05"];
<< FreeLie.m
$SeriesShowDegree = 3; $SeriesCompareDegree = 6;
<< muCalculus.m
<< "C:\\drorbn\\AcademicPensieve\\2013-04\\WKOSolution10.m"
```

```
α = MakeLieSeries[{"1", "2"}, αs];
β = MakeLieSeries[{"1", "2"}, βs];
γ = MakeCWSeries[{"1", "2"}, γs];
V = M[{1 → α, 2 → β}, γ];
κs[d_, 1] := If[OddQ[d], 0, κs[d]];
κ = MakeCWSeries[{"1"}, κs];
Unprotect[C];
C = M[{1 → MakeLieSeries[0]}, κ];
{V, C}
```

$$\left\{M\left[\left\{1 \to LS\left[0, -\frac{\overline{12}}{24}, 0\right], 2 \to LS\left[\frac{\overline{1}}{2}, -\frac{\overline{12}}{12}, 0\right]\right\}, CWS\left[0, -\frac{\widehat{12}}{48}, 0\right]\right],\right.$$
$$\left. M\left[\{1 \to LS[0, 0, 0]\}, CWS\left[0, -\frac{\widehat{11}}{96}, 0\right]\right]\right\}$$

```
{Vinv = V // dA[1] // dA[2], V ** Vinv, V ** Vinv ≡ dϵ[1] ⋃ dϵ[2]}
```

$$\left\{M\left[\left\{1 \to LS\left[0, \frac{\overline{12}}{24}, -\frac{1}{48}\overline{1\overline{12}}\right], 2 \to LS\left[-\frac{\overline{1}}{2}, \frac{\overline{12}}{12}, -\frac{1}{48}\overline{1\overline{12}}\right]\right\}, CWS\left[0, \frac{\widehat{12}}{48}, 0\right]\right],\right.$$
$$\left. M[\{1 \to LS[0, 0, 0], 2 \to LS[0, 0, 0]\}, CWS[0, 0, 0]], \text{True}\right\}$$

Φ

```
Φ = (Vinv // dσ[2, 3] // dΔ[1, 1, 2]) **
  Vinv ** (V // dσ[2, 3] // dσ[1, 2]) ** (V // dΔ[2, 2, 3])
```

$$M\left[\left\{1 \to LS\left[0, \frac{\overline{23}}{24}, 0\right], 2 \to LS\left[0, -\frac{\overline{13}}{24}, 0\right], 3 \to LS\left[0, \frac{\overline{12}}{24}, 0\right]\right\}, CWS[0, 0, 0]\right]$$

```
Φ[#]@{5} & /@ {1, 2, 3}
```

$$\left\{\text{LS}\left[0, \frac{\overline{23}}{24}, 0, -\frac{\overline{1\,\overline{1\,23}}}{1440}+\frac{\overline{7\,1\,\overline{23}\,3}}{5760}+\frac{\overline{1\,\overline{23}\,3}}{5760}-\frac{\overline{7\,2\,\overline{23}\,3}}{5760}+\right.\right.$$
$$\frac{\overline{7\,2\,\overline{23}\,3}}{5760}+\frac{1}{480}\overline{\overline{12}\,\overline{13}}-\frac{\overline{\overline{13}\,23}}{1920}+\frac{1}{640}\overline{\overline{1\,\overline{23}\,2}}-\frac{\overline{\overline{13}\,2\,2}}{1152}-\frac{\overline{\overline{13}\,3\,2}}{1152}-\frac{\overline{\overline{23}\,3\,3}}{1440}, 0\Big],$$
$$\text{LS}\Big[0, -\frac{\overline{13}}{24}, 0, \frac{\overline{1\,\overline{1\,13}}}{1440}-\frac{\overline{1\,\overline{1\,23}}}{1152}+\frac{\overline{7\,1\,\overline{23}\,3}}{1920}-\frac{1}{480}\overline{1\,\overline{\overline{13}\,2}}-\frac{\overline{1\,\overline{13}\,3}}{5760}+\frac{\overline{1\,\overline{23}\,3}}{1152}+$$
$$\frac{\overline{7\,\overline{12}\,\overline{13}}}{5760}+\frac{19\,\overline{\overline{13}\,\overline{23}}}{5760}+\frac{\overline{7\,\overline{1\,\overline{23}}\,2}}{1920}+\frac{\overline{7\,\overline{13}\,\overline{2\,2}}}{5760}+\frac{\overline{7\,\overline{13}\,\overline{3\,2}}}{5760}+\frac{\overline{\overline{13}\,\overline{3\,3}}}{1440}, 0\Big],$$
$$\text{LS}\Big[0, \frac{\overline{12}}{24}, 0, -\frac{\overline{1\,\overline{1\,2}}}{1440}+\frac{\overline{1\,\overline{1\,23}}}{5760}+\frac{\overline{7\,1\,\overline{23}\,3}}{5760}+\frac{\overline{7\,1\,\overline{12}\,2}}{5760}-\frac{\overline{1\,\overline{13}\,2}}{1440}-\frac{\overline{1\,\overline{23}\,3}}{1440}+$$
$$\left.\left.\frac{\overline{\overline{12}\,\overline{13}}}{5760}+\frac{\overline{\overline{13}\,\overline{23}}}{1440}-\frac{\overline{\overline{1\,\overline{23}}\,2}}{1152}-\frac{\overline{7\,\overline{12}\,\overline{2\,2}}}{5760}-\frac{\overline{7\,\overline{13}\,\overline{2\,2}}}{5760}-\frac{\overline{\overline{13}\,\overline{3\,2}}}{1440}, 0\Big]\right\}$$

```
Φ[W]@{6}
```

```
CWS[0, 0, 0, 0, 0, 0]
```

```
Φ[W]@{7}
```

```
CWS[0, 0, 0, 0, 0, 0, 0]
```

```
Φ[W]@{8}
```

```
$Aborted
```

Horizontal and Vertical Flips

```
{Φ ** (Φ // dσ[{1, 3} → {3, 1}]),
 (Φ ** (Φ // dσ[{1, 3} → {3, 1}])) ≡ dϵ[1] ⋃ dϵ[2] ⋃ dϵ[3]}
```

```
{M[{1 → LS[0, 0, 0], 2 → LS[0, 0, 0], 3 → LS[0, 0, 0]}, CWS[0, 0, 0]], True}
```

```
{Φ ** (Φ // dS[1] // dS[2] // dS[3]),
 Φ ** (Φ // dS[1] // dS[2] // dS[3]) ≡ dϵ[1] ⋃ dϵ[2] ⋃ dϵ[3]}
```

```
{M[{1 → LS[0, 0, 0], 2 → LS[0, 0, 0], 3 → LS[0, 0, 0]}, CWS[0, 0, 0]], True}
```

```
Φinv = Φ // dσ[{1, 3} → {3, 1}]
```

$$\text{M}\left[\left\{1 \to \text{LS}\left[0, -\frac{\overline{23}}{24}, 0\right], 2 \to \text{LS}\left[0, \frac{\overline{13}}{24}, 0\right], 3 \to \text{LS}\left[0, -\frac{\overline{12}}{24}, 0\right]\right\}, \text{CWS}[0, 0, 0]\right]$$

The Pentagon

```
{lhs = Φ ** (Φ // dσ[3, 4] // dΔ[2, 2, 3]) ** (Φ // dσ[3, 4] // dσ[2, 3] // dσ[1, 2]) ,
 rhs = (Φ // dσ[3, 4] // dσ[2, 3] // dΔ[1, 1, 2]) ** (Φ // dΔ[3, 3, 4]),
 lhs ≡ rhs
}
```

$$\left\{M\left[\left\{1 \to LS\left[0, \frac{\overline{23}}{24} + \frac{\overline{24}}{24} + \frac{\overline{34}}{24}, 0\right], 2 \to LS\left[0, -\frac{\overline{13}}{24} - \frac{\overline{14}}{24} + \frac{\overline{34}}{24}, 0\right],\right.\right.$$
$$\left.\left. 3 \to LS\left[0, \frac{\overline{12}}{24} - \frac{\overline{14}}{24} - \frac{\overline{24}}{24}, 0\right], 4 \to LS\left[0, \frac{\overline{12}}{24} + \frac{\overline{13}}{24} + \frac{\overline{23}}{24}, 0\right]\right\}, CWS[0, 0, 0]\right],$$
$$M\left[\left\{1 \to LS\left[0, \frac{\overline{23}}{24} + \frac{\overline{24}}{24} + \frac{\overline{34}}{24}, 0\right], 2 \to LS\left[0, -\frac{\overline{13}}{24} - \frac{\overline{14}}{24} + \frac{\overline{34}}{24}, 0\right],\right.\right.$$
$$\left.\left. 3 \to LS\left[0, \frac{\overline{12}}{24} - \frac{\overline{14}}{24} - \frac{\overline{24}}{24}, 0\right], 4 \to LS\left[0, \frac{\overline{12}}{24} + \frac{\overline{13}}{24} + \frac{\overline{23}}{24}, 0\right]\right\}, CWS[0, 0, 0]\right], True\right\}$$

The Hexagons

```
{lhs = Θ[1, 2, +1] // dσ[2, 3] // dΔ[1, 1, 2],
 rhs = Φ ** Θ[2, 3, +1] ** (Φinv // dσ[{2, 3} → {3, 2}]) **
   Θ[1, 3, +1] ** (Φ // dσ[{1, 2, 3} → {3, 1, 2}]),
 lhs ≡
  rhs}
```

$$\left\{M\left[\left\{1 \to LS\left[\frac{\overline{3}}{2}, \frac{\overline{13}}{8} + \frac{\overline{23}}{8}, \frac{1}{48}\overline{1\overline{13}} + \frac{1}{48}\overline{1\overline{23}} + \frac{1}{48}\overline{2\overline{23}} - \frac{1}{48}\overline{\overline{132}} - \frac{1}{96}\overline{\overline{133}} - \frac{1}{96}\overline{\overline{233}}\right],\right.\right.$$
$$2 \to LS\left[\frac{\overline{3}}{2}, \frac{\overline{13}}{8} + \frac{\overline{23}}{8}, \frac{1}{48}\overline{1\overline{13}} + \frac{1}{48}\overline{1\overline{23}} + \frac{1}{48}\overline{2\overline{23}} - \frac{1}{48}\overline{\overline{132}} - \frac{1}{96}\overline{\overline{133}} - \frac{1}{96}\overline{\overline{233}}\right],$$
$$3 \to LS\left[\frac{\overline{1}}{2} + \frac{\overline{2}}{2}, -\frac{\overline{13}}{8} - \frac{\overline{23}}{8},\right.$$
$$\left.\left.\left. -\frac{1}{96}\overline{1\overline{13}} - \frac{1}{96}\overline{1\overline{23}} - \frac{1}{96}\overline{2\overline{23}} + \frac{1}{96}\overline{\overline{132}} + \frac{1}{48}\overline{\overline{133}} + \frac{1}{48}\overline{\overline{233}}\right]\right\}, CWS[0, 0, 0]\right],$$
$$M\left[\left\{1 \to LS\left[\frac{\overline{3}}{2}, \frac{\overline{13}}{8} + \frac{\overline{23}}{8}, \frac{1}{48}\overline{1\overline{13}} + \frac{1}{48}\overline{1\overline{23}} + \frac{1}{48}\overline{2\overline{23}} - \frac{1}{48}\overline{\overline{132}} - \frac{1}{96}\overline{\overline{133}} - \frac{1}{96}\overline{\overline{233}}\right],\right.\right.$$
$$2 \to LS\left[\frac{\overline{3}}{2}, \frac{\overline{13}}{8} + \frac{\overline{23}}{8}, \frac{1}{48}\overline{1\overline{13}} + \frac{1}{48}\overline{1\overline{23}} + \frac{1}{48}\overline{2\overline{23}} - \frac{1}{48}\overline{\overline{132}} - \frac{1}{96}\overline{\overline{133}} - \frac{1}{96}\overline{\overline{233}}\right],$$
$$3 \to LS\left[\frac{\overline{1}}{2} + \frac{\overline{2}}{2}, -\frac{\overline{13}}{8} - \frac{\overline{23}}{8},\right.$$
$$\left.\left.\left. -\frac{1}{96}\overline{1\overline{13}} - \frac{1}{96}\overline{1\overline{23}} - \frac{1}{96}\overline{2\overline{23}} + \frac{1}{96}\overline{\overline{132}} + \frac{1}{48}\overline{\overline{133}} + \frac{1}{48}\overline{\overline{233}}\right]\right\}, CWS[0, 0, 0]\right], True\right\}$$

```
{lhs = Θ[1, 2, -1] // dσ[2, 3] // dΔ[1, 1, 2],
 rhs = Φ ** Θ[2, 3, -1] ** (Φinv // dσ[{2, 3} → {3, 2}]) **
   Θ[1, 3, -1] ** (Φ // dσ[{1, 2, 3} → {3, 1, 2}]),
 lhs ≡
  rhs}
```

$$\left\{M\left[\left\{1 \to LS\left[-\frac{\overline{3}}{2}, \frac{\overline{13}}{8}+\frac{\overline{23}}{8}, -\frac{1}{48}\overline{1\overline{13}}-\frac{1}{48}\overline{1\overline{23}}-\frac{1}{48}\overline{2\overline{23}}+\frac{1}{48}\overline{\overline{13}2}+\frac{1}{96}\overline{\overline{13}3}+\frac{1}{96}\overline{\overline{23}3}\right],\right.\right.\right.$$
$$2 \to LS\left[-\frac{\overline{3}}{2}, \frac{\overline{13}}{8}+\frac{\overline{23}}{8}, -\frac{1}{48}\overline{1\overline{13}}-\frac{1}{48}\overline{1\overline{23}}-\frac{1}{48}\overline{2\overline{23}}+\frac{1}{48}\overline{\overline{13}2}+\frac{1}{96}\overline{\overline{13}3}+\frac{1}{96}\overline{\overline{23}3}\right],$$
$$3 \to LS\left[-\frac{\overline{1}}{2}-\frac{\overline{2}}{2}, -\frac{\overline{13}}{8}-\frac{\overline{23}}{8},\right.$$
$$\left.\left.\left.\frac{1}{96}\overline{1\overline{13}}+\frac{1}{96}\overline{1\overline{23}}+\frac{1}{96}\overline{2\overline{23}}-\frac{1}{96}\overline{\overline{13}2}-\frac{1}{48}\overline{\overline{13}3}-\frac{1}{48}\overline{\overline{23}3}\right]\right\}, \text{CWS}[0, 0, 0]\right],$$
$$M\left[\left\{1 \to LS\left[-\frac{\overline{3}}{2}, \frac{\overline{13}}{8}+\frac{\overline{23}}{8}, -\frac{1}{48}\overline{1\overline{13}}-\frac{1}{48}\overline{1\overline{23}}-\frac{1}{48}\overline{2\overline{23}}+\frac{1}{48}\overline{\overline{13}2}+\frac{1}{96}\overline{\overline{13}3}+\frac{1}{96}\overline{\overline{23}3}\right],\right.\right.$$
$$2 \to LS\left[-\frac{\overline{3}}{2}, \frac{\overline{13}}{8}+\frac{\overline{23}}{8}, -\frac{1}{48}\overline{1\overline{13}}-\frac{1}{48}\overline{1\overline{23}}-\frac{1}{48}\overline{2\overline{23}}+\frac{1}{48}\overline{\overline{13}2}+\frac{1}{96}\overline{\overline{13}3}+\frac{1}{96}\overline{\overline{23}3}\right],$$
$$3 \to LS\left[-\frac{\overline{1}}{2}-\frac{\overline{2}}{2}, -\frac{\overline{13}}{8}-\frac{\overline{23}}{8},\right.$$
$$\left.\left.\left.\frac{1}{96}\overline{1\overline{13}}+\frac{1}{96}\overline{1\overline{23}}+\frac{1}{96}\overline{2\overline{23}}-\frac{1}{96}\overline{\overline{13}2}-\frac{1}{48}\overline{\overline{13}3}-\frac{1}{48}\overline{\overline{23}3}\right]\right\}, \text{CWS}[0, 0, 0]\right], \text{True}\right\}$$