

Pensieve header: Simplifying the WKO equations written in MGA language.

Simplifying the Equations

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-05"];
<< FreeLie.m
$SeriesShowDegree = 3; $SeriesCompareDegree = 6;
<< muCalculus.m

α = RandomLieSeries[{"1", "2"}];
β = RandomLieSeries[{"1", "2"}];
γ = RandomCWSeries[{"1", "2"}];
V = M[{1 → α, 2 → β}, γ];
κ = RandomCWSeries[{"1"}];
Unprotect[C]; C = M[{1 → MakeLieSeries[0]}, κ];
{V, C}

{M[{1 → LS[2 1̄ + 2 2̄, -2 1̄2̄, - $\frac{2}{3}$  1̄1̄2̄ -  $\frac{3}{2}$  1̄2̄2̄], 2 → LS[1̄, 2 1̄2̄, -1̄1̄2̄ + 1̄2̄2̄]},
  CWS[1̄ - 2 2̄, - $\frac{3}{2}$  1̄1̄ +  $\frac{12}{2}$  +  $\frac{3}{2}$  2̄2̄, -1̄1̄1̄ -  $\frac{5}{6}$  1̄1̄2̄ -  $\frac{122}{2}$  -  $\frac{222}{3}$ ]],
  M[{1 → LS[0, 0, 0]}, CWS[2 1̄,  $\frac{3}{2}$  1̄1̄,  $\frac{3}{2}$  1̄1̄1̄]]}]

HardR4[d_, V_] :=
Module[{lhs = R+[2, 3] ** R+[1, 3] ** V, rhs = V ** (R+[1, 3] // dΔ[1, 1, 2])},
  {lhs[1]@d - rhs[1]@d, lhs[2]@d - rhs[2]@d,
   lhs[3]@d - rhs[3]@d, lhs[W]@d - rhs[W]@d}
];

TwistEq[d_, V_] :=
Module[{lhs = V ** θ[1, 2], rhs = R+[1, 2] ** (V // dσ[{1, 2} → {2, 1}])},
  {lhs[1]@d - rhs[1]@d, lhs[2]@d - rhs[2]@d, lhs[W]@d - rhs[W]@d}
];

UnitarityEq[d_, V_] :=
Module[{lhs = V ** (V // dA[1] // dA[2]), rhs = dε[1] ∪ dε[2]},
  {lhs[1]@d - rhs[1]@d, lhs[2]@d - rhs[2]@d, lhs[W]@d - rhs[W]@d}
];

CapEq[d_, V_, C_] :=
Module[{lhs = V ** (C // dΔ[1, 1, 2]) // dc[1] // dc[2], rhs = C ∪ (C // dσ[1, 2])},
  {lhs[W]@d - rhs[W]@d}
];

VerticalFlipEq[d_, V_] :=
Module[{lhs = V ** (V // dS[1] // dS[2]), rhs = R+[1, 2]},
  {lhs[1]@d - rhs[1]@d, lhs[2]@d - rhs[2]@d, lhs[W]@d - rhs[W]@d}
];

```

L1 = R⁺[2, 3] ** R⁺[1, 3] ** V

$$M\left[\left\{1 \rightarrow LS\left[2\overline{1} + 2\overline{2}, -2\overline{12}, -\frac{2}{3}\overline{112} - \frac{3}{2}\overline{122}\right],\right.\right.$$

$$2 \rightarrow LS\left[\overline{1}, 2\overline{12}, -\overline{112} + \overline{122}\right], 3 \rightarrow LS\left[\overline{1} + \overline{2}, -\frac{3\overline{12}}{2}, \frac{1}{12}\overline{112} + \frac{61}{12}\overline{122}\right]\left.\right\},$$

$$CWS\left[\widehat{1} - 2\widehat{2}, -\frac{3\widehat{11}}{2} + \frac{\widehat{12}}{2} + \frac{3\widehat{22}}{2}, -\widehat{111} - \frac{5\widehat{112}}{6} - \frac{\widehat{122}}{2} - \frac{\widehat{222}}{3}\right]]$$

L1 = ρ⁺[2, 3] ∪ ρ⁺[1, 4] ∪ (V // tσ[1, 5] // tσ[2, 6]) // hm[3, 4, 3] // tha[1, 1] // tm[1, 5, 1] // tha[2, 2] // tm[2, 6, 2]

$$M\left[\left\{1 \rightarrow LS\left[2\overline{1} + 2\overline{2}, -2\overline{12}, -\frac{2}{3}\overline{112} - \frac{3}{2}\overline{122}\right],\right.\right.$$

$$2 \rightarrow LS\left[\overline{1}, 2\overline{12}, -\overline{112} + \overline{122}\right], 3 \rightarrow LS\left[\overline{1} + \overline{2}, -\frac{3\overline{12}}{2}, \frac{1}{12}\overline{112} + \frac{61}{12}\overline{122}\right]\left.\right\},$$

$$CWS\left[\widehat{1} - 2\widehat{2}, -\frac{3\widehat{11}}{2} + \frac{\widehat{12}}{2} + \frac{3\widehat{22}}{2}, -\widehat{111} - \frac{5\widehat{112}}{6} - \frac{\widehat{122}}{2} - \frac{\widehat{222}}{3}\right]]$$

L1 ≡ L1

True

r1 = V ** (R⁺[1, 3] // dΔ[1, 1, 2])

$$M\left[\left\{1 \rightarrow LS\left[2\overline{1} + 2\overline{2}, -2\overline{12}, -\frac{2}{3}\overline{112} - \frac{3}{2}\overline{122}\right], 2 \rightarrow LS\left[\overline{1}, 2\overline{12}, -\overline{112} + \overline{122}\right],\right.\right.$$

$$3 \rightarrow LS\left[\overline{1} + \overline{2}, 0, 0\right]\left.\right\}, CWS\left[\widehat{1} - 2\widehat{2}, -\frac{3\widehat{11}}{2} + \frac{\widehat{12}}{2} + \frac{3\widehat{22}}{2}, -\widehat{111} - \frac{5\widehat{112}}{6} - \frac{\widehat{122}}{2} - \frac{\widehat{222}}{3}\right]]$$

R1 = V ∪ (ρ⁺[4, 3] // tΔ[4, 4, 5]) // tm[1, 4, 1] // tm[2, 5, 2]

$$M\left[\left\{1 \rightarrow LS\left[2\overline{1} + 2\overline{2}, -2\overline{12}, -\frac{2}{3}\overline{112} - \frac{3}{2}\overline{122}\right], 2 \rightarrow LS\left[\overline{1}, 2\overline{12}, -\overline{112} + \overline{122}\right],\right.\right.$$

$$3 \rightarrow LS\left[\overline{1} + \overline{2}, 0, 0\right]\left.\right\}, CWS\left[\widehat{1} - 2\widehat{2}, -\frac{3\widehat{11}}{2} + \frac{\widehat{12}}{2} + \frac{3\widehat{22}}{2}, -\widehat{111} - \frac{5\widehat{112}}{6} - \frac{\widehat{122}}{2} - \frac{\widehat{222}}{3}\right]]$$

r1 ≡ R1

True