

$\rho^+[u, x], \rho^+[v, y]$

$\{L[Hs[] Ts[] + Hs[x] (-1 + t_u) Ts[u], \{x \rightarrow t_u\}],$
 $L[Hs[] Ts[] + Hs[y] (-1 + t_v) Ts[v], \{y \rightarrow t_v\}]\}$

$\rho^+[u, x] \rho^+[v, y]$

$L[Hs[] Ts[] - Hs[x] Ts[u] + Hs[x] t_u Ts[u] - Hs[y] Ts[v] + Hs[y] t_v Ts[v] + Hs[x, y] Ts[u, v] -$
 $Hs[x, y] t_u Ts[u, v] - Hs[x, y] t_v Ts[u, v] + Hs[x, y] t_u t_v Ts[u, v], \{x \rightarrow t_u, y \rightarrow t_v\}]$

$\Lambda 1 = \rho^+[u, x] \rho^+[v, y] \rho^+[w, z]$

$L[Hs[] Ts[] - Hs[x] Ts[u] + Hs[x] t_u Ts[u] - Hs[y] Ts[v] + Hs[y] t_v Ts[v] - Hs[z] Ts[w] +$
 $Hs[z] t_w Ts[w] + Hs[x, y] Ts[u, v] - Hs[x, y] t_u Ts[u, v] - Hs[x, y] t_v Ts[u, v] +$
 $Hs[x, y] t_u t_v Ts[u, v] + Hs[x, z] Ts[u, w] - Hs[x, z] t_u Ts[u, w] -$
 $Hs[x, z] t_w Ts[u, w] + Hs[x, z] t_u t_w Ts[u, w] + Hs[y, z] Ts[v, w] -$
 $Hs[y, z] t_v Ts[v, w] - Hs[y, z] t_w Ts[v, w] + Hs[y, z] t_v t_w Ts[v, w] -$
 $Hs[x, y, z] Ts[u, v, w] + Hs[x, y, z] t_u Ts[u, v, w] + Hs[x, y, z] t_v Ts[u, v, w] -$
 $Hs[x, y, z] t_u t_v Ts[u, v, w] + Hs[x, y, z] t_w Ts[u, v, w] - Hs[x, y, z] t_u t_w Ts[u, v, w] -$
 $Hs[x, y, z] t_v t_w Ts[u, v, w] + Hs[x, y, z] t_u t_v t_w Ts[u, v, w], \{x \rightarrow t_u, y \rightarrow t_v, z \rightarrow t_w\}]$

$\Lambda 1 // \text{tm}[u, v, u] // \text{tm}[u, w, u]$

$L[Hs[] Ts[] - Hs[x] Ts[u] - Hs[y] Ts[u] - Hs[z] Ts[u] +$
 $Hs[x] t_u Ts[u] + Hs[y] t_u Ts[u] + Hs[z] t_u Ts[u], \{x \rightarrow t_u, y \rightarrow t_u, z \rightarrow t_u\}]$

$\Lambda 1 // \text{tm}[v, w, v] // \text{tm}[u, v, u]$

$L[Hs[] Ts[] - Hs[x] Ts[u] - Hs[y] Ts[u] - Hs[z] Ts[u] +$
 $Hs[x] t_u Ts[u] + Hs[y] t_u Ts[u] + Hs[z] t_u Ts[u], \{x \rightarrow t_u, y \rightarrow t_u, z \rightarrow t_u\}]$

$\Lambda 1 // \text{hm}[x, y, x] // \text{hm}[x, z, x]$

$L[Hs[] Ts[] - Hs[x] Ts[u] + Hs[x] t_u Ts[u] - Hs[x] t_u Ts[v] +$
 $Hs[x] t_u t_v Ts[v] - Hs[x] t_u t_v Ts[w] + Hs[x] t_u t_v t_w Ts[w], \{x \rightarrow t_u t_v t_w\}]$

$\Lambda 1 // \text{hm}[y, z, y] // \text{hm}[x, y, x]$

$L[Hs[] Ts[] - Hs[x] Ts[u] + Hs[x] t_u Ts[u] - Hs[x] t_u Ts[v] +$
 $Hs[x] t_u t_v Ts[v] - Hs[x] t_u t_v Ts[w] + Hs[x] t_u t_v t_w Ts[w], \{x \rightarrow t_u t_v t_w\}]$

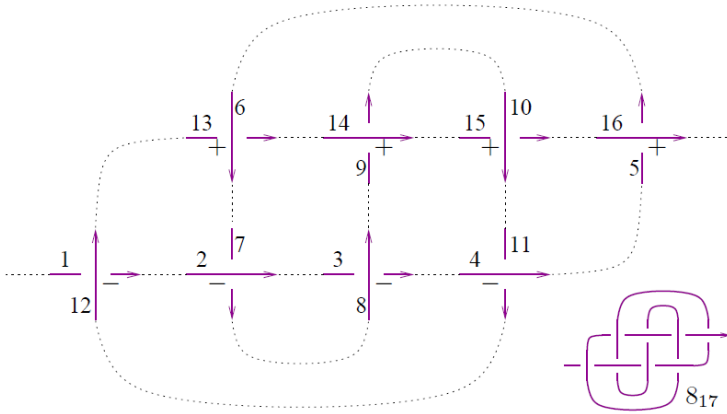
$\{t1 = \rho^+[u, x] \rho^+[v, y] \rho^+[w, z] // \text{tm}[v, w, v] // \text{hm}[x, y, x] // \text{tha}[u, z],$

$t2 = \rho^+[v, x] \rho^+[w, z] \rho^+[u, y] // \text{tm}[v, w, v] // \text{hm}[x, y, x],$

$t1 == t2\}$

$\{L[Hs[] Ts[] - Hs[x] t_v Ts[u] + Hs[x] t_u t_v Ts[u] - Hs[x] Ts[v] - Hs[z] Ts[v] +$
 $Hs[x] t_v Ts[v] + Hs[z] t_v Ts[v] + Hs[x, z] t_v Ts[u, v] - Hs[x, z] t_u t_v Ts[u, v] -$
 $Hs[x, z] t_v^2 Ts[u, v] + Hs[x, z] t_u t_v^2 Ts[u, v], \{x \rightarrow t_u t_v, z \rightarrow t_v\}],$

$L[Hs[] Ts[] - Hs[x] t_v Ts[u] + Hs[x] t_u t_v Ts[u] - Hs[x] Ts[v] - Hs[z] Ts[v] +$
 $Hs[x] t_v Ts[v] + Hs[z] t_v Ts[v] + Hs[x, z] t_v Ts[u, v] - Hs[x, z] t_u t_v Ts[u, v] -$
 $Hs[x, z] t_v^2 Ts[u, v] + Hs[x, z] t_u t_v^2 Ts[u, v], \{x \rightarrow t_u t_v, z \rightarrow t_v\}], \text{True}\}$



$$\Lambda_1 = R^-[12, 1] R^-[2, 7] R^-[8, 3] R^-[4, 11] R^+[16, 5] R^+[6, 13] R^+[14, 9] R^+[10, 15]$$

A very large output was generated. Here is a sample of it:

```
L[Hs[] Ts[] - Hs[7] Ts[2] + <<9837>> +
(Hs[1, 3, 5, 7, 9, 11, 13, 15] t6 t10 t14 t16 Ts[2, 4, 6, 8, 10, 12, 14, 16]) /
(t2 t4 t8 t12), {1 → 1/t12, 2 → 1, 3 → 1/t8, 4 → 1,
5 → t16, <<6>>, 12 → 1, 13 → t6, 14 → 1, 15 → t10, 16 → 1}]
```

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$$\Lambda_2 = \Lambda_1; \text{Do}[\Lambda_2 = \Lambda_2 // \text{dm}[1, k, 1], \{k, 2, 16\}]; \Lambda_2$$

```
L[11 Hs[] Ts[] - Hs[] Ts[] / t13 + 4 Hs[] Ts[] / t12 -
8 Hs[] Ts[] / t1 - 8 Hs[] t1 Ts[] + 4 Hs[] t12 Ts[] - Hs[] t13 Ts[], {1 → 1}]
```

Coefficient[Λ2[[1]], Hs[] Ts[]]

$$11 - \frac{1}{t_1^3} + \frac{4}{t_1^2} - \frac{8}{t_1} - 8 t_1 + 4 t_1^2 - t_1^3$$

<< KnotTheory`

Loading KnotTheory` version of February 5, 2013, 3:48:46.4762.
Read more at <http://katlas.org/wiki/KnotTheory>.

Alexander[Knot[8, 17]][t]

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$11 - \frac{1}{t^3} + \frac{4}{t^2} - \frac{8}{t} - 8 t + 4 t^2 - t^3$$

```
Λ4 = Λ1; Do[Λ4 = Λ4 // dm[k, k - 1, k - 1], {k, 16, 8, -1}]; Λ4
```

A very large output was generated. Here is a sample of it:

$$L \left[\frac{Hs[] Ts[] - \frac{Hs[] t_6 Ts[]}{t_4} + \ll 1033 \gg + \frac{Hs[1, 3, 5, 7] t_6^2 t_7^5 Ts[2, 4, 6, 7]}{t_2 t_4}, \left\{ 1 \rightarrow \frac{1}{t_7}, 2 \rightarrow 1, 3 \rightarrow \frac{1}{t_7}, 4 \rightarrow 1, 5 \rightarrow t_7, 6 \rightarrow 1, 7 \rightarrow \frac{t_6 t_7^2}{t_2 t_4} \right\} \right]$$

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```
{u, v, w} = {1, 2, 3};
t1 = Λ4 // tm[u, v, u] // tm[u, w, u];
t2 = Λ4 // tm[v, w, v] // tm[u, v, u];
t1 == t2
{True}
```

```
{x, y, z} = {1, 2, 3};
(Λ4 // hm[x, y, x] // hm[x, z, x]) == (Λ4 // hm[y, z, y] // hm[x, y, x])
True
```

```
{a, b, c} = {1, 2, 3};
(Λ4 // dm[a, b, a] // dm[a, c, a]) == (Λ4 // dm[b, c, b] // dm[a, b, a])
True
```

```
Λ4 // dm[a, b, a] // dm[a, c, a]
```

A very large output was generated. Here is a sample of it:

$$L \left[\frac{Hs[] Ts[]}{t_1} + \frac{Hs[] t_6 Ts[]}{t_4} - \frac{2 Hs[] t_6 Ts[]}{t_1 t_4} + \ll 893 \gg + \frac{Hs[1, 5, 7] t_6 t_7^5 Ts[4, 6, 7]}{t_1^2 t_4} - \frac{Hs[1, 5, 7] t_6 t_7^5 Ts[4, 6, 7]}{t_1^2 t_4} - \frac{Hs[1, 5, 7] t_6^2 t_7^5 Ts[4, 6, 7]}{t_1^2 t_4} + \frac{Hs[1, 5, 7] t_6^2 t_7^5 Ts[4, 6, 7]}{t_1^2 t_4}, \left\{ 1 \rightarrow \frac{1}{t_7}, 4 \rightarrow 1, 5 \rightarrow t_7, 6 \rightarrow 1, 7 \rightarrow \frac{t_6 t_7^2}{t_1 t_4} \right\} \right]$$

Show Less Show More Show Full Output Set Size Limit...

```
{R+ [1, 2] // B, R- [1, 2] // B}
```

$$\left\{ \begin{pmatrix} 1 & h[1] & h[2] \\ t[1] & 0 & -1 + T_1 \\ 1 + \Sigma/\omega & 1 & T_1 \end{pmatrix}, \begin{pmatrix} 1 & h[1] & h[2] \\ t[1] & 0 & -\frac{-1 + T_1}{T_1} \\ 1 + \Sigma/\omega & 1 & \frac{1}{T_1} \end{pmatrix} \right\}$$

```
R+ [1, 2]
```

```
L[Hs[] Ts[] - Hs[2] Ts[1] + Hs[2] t1 Ts[1], {1 → 1, 2 → t1}]
```

$\beta_1 = B[\Delta_1]$

$$\begin{pmatrix} 1 & h[1] & h[2] & h[3] & h[4] & h[5] & h[6] & h[7] & h[8] & h[9] & h[10] & h[11] & h[12] \\ t[2] & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{-1+T_2}{T_2} & 0 & 0 & 0 & 0 & 0 \\ t[4] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{-1+T_4}{T_4} & 0 \\ t[6] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t[8] & 0 & 0 & -\frac{-1+T_8}{T_8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t[10] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t[12] & -\frac{-1+T_{12}}{T_{12}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t[14] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 + T_{14} & 0 & 0 & 0 \\ t[16] & 0 & 0 & 0 & 0 & -1 + T_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1+\Sigma/\omega & \frac{1}{T_{12}} & 1 & \frac{1}{T_8} & 1 & T_{16} & 1 & \frac{1}{T_2} & 1 & T_{14} & 1 & \frac{1}{T_4} & 1 \end{pmatrix}$$

$\Delta_1 // \mathbf{dm}[1, 2, 1] // \mathbf{dm}[1, 3, 1] // \mathbf{dm}[1, 4, 1] // \mathbf{dm}[1, 5, 1] // \mathbf{dm}[1, 6, 1] //$
 $\mathbf{dm}[1, 7, 1] // \mathbf{dm}[1, 8, 1] // \mathbf{dm}[1, 9, 1] // \mathbf{dm}[1, 10, 1] //$
 $\mathbf{dm}[1, 11, 1] // \mathbf{dm}[1, 12, 1] // \mathbf{dm}[1, 13, 1] // \mathbf{dm}[1, 14, 1] // B$

$$\begin{pmatrix} -\frac{1-2 T_1+T_1^2-T_1^3-2 T_{16}+3 T_1 T_{16}-2 T_1^2 T_{16}+T_1^3 T_{16}}{T_1^2} & h[1] \\ t[1] & \frac{(-1+T_1) (-T_1+2 T_1^2-4 T_1^3+2 T_1^4-T_1^5+T_{16}-T_1^2 T_{16}+3 T_1^3 T_{16}-2 T_1^4 T_{16}+T_1^5 T_{16}-2 T_{16}^2+T_1 T_{16}^2-T_1^2 T_{16}^2)}{T_1^4} \\ t[16] & -\frac{(-1+4 T_1-8 T_1^2+7 T_1^3-4 T_1^4+T_1^5) (-1+T_{16})}{T_1^2} \\ 1+\Sigma/\omega & \frac{T_{16}}{T_1^2} \end{pmatrix}$$

$\Delta_1 // B // \mathbf{dm}[1, 2, 1] // \mathbf{dm}[1, 3, 1] // \mathbf{dm}[1, 4, 1] // \mathbf{dm}[1, 5, 1] // \mathbf{dm}[1, 6, 1] //$
 $\mathbf{dm}[1, 7, 1] // \mathbf{dm}[1, 8, 1] // \mathbf{dm}[1, 9, 1] // \mathbf{dm}[1, 10, 1] // \mathbf{dm}[1, 11, 1] //$
 $\mathbf{dm}[1, 12, 1] // \mathbf{dm}[1, 13, 1] // \mathbf{dm}[1, 14, 1] // \mathbf{bbCollect}$

$$\begin{pmatrix} -\frac{1-2 T_1+T_1^2-T_1^3-2 T_{16}+3 T_1 T_{16}-2 T_1^2 T_{16}+T_1^3 T_{16}}{T_1^2} & h[1] \\ t[1] & \frac{(-1+T_1) (-T_1+2 T_1^2-4 T_1^3+2 T_1^4-T_1^5+T_{16}-T_1^2 T_{16}+3 T_1^3 T_{16}-2 T_1^4 T_{16}+T_1^5 T_{16}-2 T_{16}^2+T_1 T_{16}^2-T_1^2 T_{16}^2)}{T_1^4} \\ t[16] & -\frac{(-1+4 T_1-8 T_1^2+7 T_1^3-4 T_1^4+T_1^5) (-1+T_{16})}{T_1^2} \\ 1+\Sigma/\omega & \frac{T_{16}}{T_1^2} \end{pmatrix}$$

```

n = 6;
ops = Composition@@Reverse@Table[dm[1, k, 1], {k, 2, n}];
t1 = A1 // ops // B;
t2 = A1 // B // ops // bbCollect;
{t1, t2, t1 == t2}

```

$$\left\{ \begin{array}{l}
 1 \quad h[1] \quad h[7] \quad h[8] \quad h[9] \quad h[10] \quad h[11] \quad h[12] \quad h[13] \quad h[14] \quad h[16] \\
 t[1] \quad 0 \quad -\frac{(-1+T_1) T_{16}}{T_1 T_8} \quad 0 \quad 0 \quad 0 \quad -\frac{(-1+T_1) T_{16}}{T_1} \quad 0 \quad -1 + T_1 \quad 0 \quad 0 \\
 t[8] \quad -\frac{-1+T_8}{T_8 T_{12}} \quad -\frac{(-1+T_1) (-1+T_8)}{T_1 T_8} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
 t[10] \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1 + T_1 \\
 t[12] \quad -\frac{-1+T_{12}}{T_{12}} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
 t[14] \quad 0 \quad 0 \quad 0 \quad -1 + T_{14} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
 t[16] \quad \frac{-1+T_{16}}{T_8 T_{12}} \quad \frac{(-1+T_1) (-1+T_{16})}{T_1 T_8} \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{(-1+T_1) (-1+T_{16})}{T_1} \quad 0 \quad 0 \quad 0 \quad 0 \\
 1+\Sigma/\omega \quad \frac{T_{16}}{T_8 T_{12}} \quad \frac{1}{T_1} \quad 1 \quad T_{14} \quad 1 \quad \frac{1}{T_1} \quad 1 \quad T_1 \quad 1 \quad T_1
 \end{array} \right.$$

```

n = 6;
ops = Composition@@Reverse@Join[Table[dm[1, k, 1], {k, 2, n}], {tha[1, 7]}];
t1 = A1 // ops // B;
t2 = A1 // B // ops // bbCollect;
{t1, t2, t1 == t2}

```

$$\left\{ \begin{array}{l}
 \frac{T_1 T_8 + T_{16} - T_1 T_{16}}{T_1 T_8} \quad h[1] \quad h[7] \quad h[8] \quad h[9] \quad h[10] \quad h[16] \\
 t[1] \quad 0 \quad -\frac{(-1+T_1) T_{16}}{T_1^2 T_8} \quad 0 \quad 0 \quad 0 \quad -\frac{(-1+T_1) T_{16}}{T_1} \\
 t[8] \quad -\frac{(-1+T_8) (T_1 T_8 + T_{16} - T_1 T_{16})}{T_1 T_8^2 T_{12}} \quad -\frac{(-1+T_1) (-1+T_8)}{T_1 T_8} \quad 0 \quad 0 \quad 0 \quad -\frac{(-1+T_1) T_{16}}{T_1} \\
 t[10] \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
 t[12] \quad -\frac{(-1+T_{12}) (T_1 T_8 + T_{16} - T_1 T_{16})}{T_1 T_8 T_{12}} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
 t[14] \quad 0 \quad 0 \quad 0 \quad \frac{(-1+T_{14}) (T_1 T_8 + T_{16} - T_1 T_{16})}{T_1 T_8} \quad 0 \quad 0 \\
 t[16] \quad \frac{(-1+T_{16}) (T_1 T_8 + T_{16} - T_1 T_{16})}{T_1 T_8^2 T_{12}} \quad \frac{(-1+T_1) (-1+T_{16})}{T_1 T_8} \quad 0 \quad 0 \quad 0 \quad \frac{(-1+T_1) T_{16}}{T_1} \\
 1+\Sigma/\omega \quad \frac{T_{16}}{T_8 T_{12}} \quad \frac{1}{T_1} \quad 1 \quad T_{14} \quad 1 \quad 0
 \end{array} \right.$$