The "other" variations of C and RC

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Property 9.6. ad_u *is the infinitesimal version of both* C_u *and* RC_u*. Namely, if* $\delta\gamma$ *is an infinitesimal, then* $C_u^{\delta\gamma} = RC_u^{\delta\gamma} = 1 + \mathrm{ad}_u\{\delta\gamma\}$.

We omit the easy proof of this property and move on to δC_u^γ and δRC_u^γ:

Lemma 9.7.

$$\delta C_u^\gamma = \mathrm{ad}_u\left\{\delta\gamma \,/\!/\, \frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma} \,/\!/\, RC_u^{-\gamma}\right\} /\!/\, C_u^\gamma$$

and

$$\delta RC_u^\gamma = RC_u^\gamma \,/\!/\, \mathrm{ad}_u\left\{\delta\gamma \,/\!/\, \frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma} \,/\!/\, RC_u^\gamma\right\}.$$

Proof. Substitute α and $\delta\beta$ into Equation (12) and get $RC_u^{\mathrm{bch}(\alpha,\delta\beta)} = RC_u^\alpha \,/\!/\, RC_u^{\delta\beta /\!/ RC_u^\alpha}$, and hence using Property 9.6 for the infinitesimal $\delta\beta \,/\!/\, RC_u^\alpha$ and Lemma 9.4 with $\delta\alpha = \beta = 0$ on $\mathrm{bch}(\alpha,\delta\beta)$,

$$RC_u^{\alpha+(\delta\beta /\!/ \frac{\mathrm{ad}\,\alpha}{1-e^{-\mathrm{ad}\,\alpha}})} = RC_u^\alpha + RC_u^\alpha \,/\!/\, \mathrm{ad}_u\{\delta\beta \,/\!/\, RC_u^\alpha\}.$$

Now replacing $\alpha \to \gamma$ and $\delta\beta \to \delta\gamma \,/\!/\, \frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}$, we get the equation for δRC_u^γ. The equation for δC_U^γ now follows by taking the variation of $C_u^\gamma \,/\!/\, RC_u^{-\gamma} = Id$.

Substitute $\delta\alpha$ & β into $RC_u^{bch(\alpha,\beta)} = RC_u^\alpha /\!/ RC_u^{\beta /\!/ RC_u^\alpha}$,

get

$$RC_u^{\beta+\delta\alpha /\!/ e^{-\mathrm{ad}\beta} /\!/ \frac{\mathrm{ad}\beta}{1-e^{-\mathrm{ad}\beta}}} = (1+\mathrm{ad}_u^{\delta\alpha}) /\!/ RC_u^{\beta+\mathrm{ad}_u^{\delta\alpha}\beta}$$

seems stuck.