

Pensieve header: Experiments with tder.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-04"];
```

```
<< CheatSheetJ-Verification.m;
```

```
$SeriesCompareDegree = 10;
```

```
{α = MakeLieSeries[u], γ = MakeLieSeries[u + v + b[u, v]]}
```

```
{⟨t⟩, ⟨u⟩, ⟨v⟩, ⟨w⟩}
```

$$\text{LS}\left[-t - 2u - v, \overline{tu} - \frac{3\overline{tv}}{2} + \frac{3\overline{uv}}{2}, -2\overline{ttu} + \frac{5}{3}\overline{ttv} - \frac{5}{6}\overline{uuv} + \frac{1}{3}\overline{tuu} - \overline{tvu} + \frac{7}{6}\overline{tvv} - \frac{2}{3}\overline{uvv}\right]$$

$$\text{LS}\left[-2t - 2u - 2v, \frac{\overline{tu}}{2} + \overline{tv} - \overline{uv}, -\frac{1}{3}\overline{ttu} - \frac{11}{6}\overline{tuv} + \frac{7}{6}\overline{uuv} + \frac{1}{6}\overline{tuu} + \frac{4}{3}\overline{tvu} + \frac{5}{3}\overline{tvv} + \frac{11}{6}\overline{uvv}\right]$$

$$\text{LS}\left[t - v, \frac{3\overline{tu}}{2} + \frac{3\overline{tv}}{2} - \frac{\overline{uv}}{2}, -2\overline{ttu} - \frac{5}{6}\overline{ttv} + \frac{2}{3}\overline{tuv} + \frac{7}{6}\overline{uuv} + 2\overline{tuu} - \frac{7}{6}\overline{tvu} - \frac{2}{3}\overline{tvv} + \frac{2}{3}\overline{uvv}\right]$$

```
{LS[u, 0, 0], LS[u + v, uv̄, 0]}
```

$$e^{\text{ad}_u\{\gamma\}} = C_u^?$$

```
lhs = Module[{αk}, {αk = α} + Sum[αk = (αk // ad_u[γ]), {k, 1, 10} / k!];
```

```
lhs@{6}
```

$$\text{LS}\left[\langle u \rangle, -\langle uv \rangle, -\langle uuv \rangle + \frac{\langle uvv \rangle}{2}, \langle uuvv \rangle - \frac{\langle uvvv \rangle}{6}, \frac{\langle uuvvv \rangle}{2} + \frac{\langle uuvuv \rangle}{2} - \frac{\langle uuvvvv \rangle}{2} - \frac{\langle uvuvvv \rangle}{2} + \frac{\langle uvvvvv \rangle}{24}, -\frac{\langle uuuvvv \rangle}{2} - \langle uuvuvv \rangle - \frac{\langle uuvvvv \rangle}{2} + \frac{\langle uuuvvvv \rangle}{6} + \frac{\langle uvuvvvv \rangle}{3} - \frac{\langle uvvvvvv \rangle}{120}\right]$$

```
t[1] = γ;
```

```
t[k_] /; k > 1 := t[k] = t[k-1] // ad_u[γ];
```

```
Table[t[k]@k, {k, 7}]
```

```
{⟨u⟩ + ⟨v⟩, -⟨uv⟩, ⟨uvv⟩, -⟨uvvv⟩, ⟨uvvvv⟩, -⟨uvvvvv⟩, ⟨uvvvvvv⟩}
```

```
β[1] = t[1]
```

```
LS[u + v, uv̄, 0]
```

```
(α // CC_u[β[1]]) - lhs
```

$$\text{LS}\left[0, 0, -\frac{1}{2}\overline{uuv}\right]$$

$$\beta[2] = \beta[1] + \frac{1}{2} t[2];$$

$((\alpha // CC_u[\beta[2]]) - lhs) \equiv \text{MakeLieSeries}[0]$

$$\frac{\langle uuuv \rangle}{12} + \frac{\langle uuvv \rangle}{12} = 0$$

$$\beta[3] = \beta[2] + a t[3] + b b[t[1], t[2]];$$

$e[3] = \text{Collect}[\text{First}[(\alpha // CC_u[\beta[3]]) - lhs) \equiv \text{MakeLieSeries}[0]], _LW]$

$$\left(\frac{1}{12} + b\right) \langle uuuv \rangle + \left(\frac{1}{12} - a - b\right) \langle uuvv \rangle$$

$\beta[3]$

$$\text{LS}\left[u + v, \frac{\overline{uv}}{2}, -\frac{1}{2} \overline{u\overline{uv}} - b \overline{u\overline{uv}} - \frac{1}{2} \overline{u\overline{v\overline{v}}} + a \overline{u\overline{v\overline{v}}} + b \overline{u\overline{v\overline{v}}}\right]$$

$$\beta[3] = \beta[2] + \frac{1}{6} t[3] - \frac{1}{12} b[t[1], t[2]];$$

$e[3] = \text{Collect}[\text{First}[(\alpha // CC_u[\beta[3]]) - lhs) \equiv \text{MakeLieSeries}[0]], _LW]$

$$-\frac{\langle uuuvv \rangle}{24}$$

$$\beta[4] = \beta[3] + a t[4] + b b[t[1], t[3]];$$

$e[4] = \text{Collect}[\text{First}[(\alpha // CC_u[\beta[4]]) - lhs) \equiv \text{MakeLieSeries}[0]], _LW]$

$$\left(-\frac{1}{24} - b\right) \langle uuuvv \rangle + (a + b) \langle uuvvv \rangle$$

$$\beta[4] = \beta[3] + \frac{1}{24} t[4] - \frac{1}{24} b[t[1], t[3]];$$

$e[4] = \text{Collect}[\text{First}[(\alpha // CC_u[\beta[4]]) - lhs) \equiv \text{MakeLieSeries}[0]], _LW]$

$$-\frac{\langle uuuuvv \rangle}{720} + \frac{\langle uuuuvv \rangle}{180} + \frac{\langle uuuvuv \rangle}{360} + \frac{\langle uuuvvv \rangle}{180} + \frac{\langle uuvuvv \rangle}{120} - \frac{\langle uuvvvv \rangle}{720}$$

$$\beta[5] = \beta[4] + \frac{1}{5!} (a t[5] + b b[t[1], t[4]] + c b[t[2], t[3]] + d b[t[1], b[t[1], t[3]]] +$$

$$e b[t[1], b[t[1], b[t[1], t[2]]]] + f b[t[2], b[t[1], t[2]]]);$$

$e[5] = \text{Collect}[6! \text{First}[(\alpha // CC_u[\beta[5]]) - lhs) \equiv \text{MakeLieSeries}[0]], _LW, \text{Expand}]$

$$\begin{aligned} &(-1 + 6e) \langle uuuuvv \rangle + (4 - 6d - 18e) \langle uuuvuv \rangle + \\ &(2 + 6e + 6f) \langle uuuvvv \rangle + (4 + 6b + 12d + 18e) \langle uuvuvv \rangle + \\ &(6 + 6c + 6d + 12e + 6f) \langle uuvvvv \rangle + (-1 - 6a - 6b - 6d - 6e) \langle uuvvvv \rangle \end{aligned}$$

$$\beta[5] = \beta[4] + \frac{1}{5!} \left(t[5] - \frac{3}{2} b[t[1], t[4]] - b[t[2], t[3]] + \frac{1}{6} b[t[1], b[t[1], t[3]]] + \frac{1}{6} b[t[1], b[t[1], b[t[1], t[2]]] - \frac{1}{2} b[t[2], b[t[1], t[2]]] \right);$$

`e[5] = Collect[6! First[(α // CCu[β [5]]) - lhs] ≡ MakeLieSeries[0]], _LW, Expand]`

$$\frac{\langle uuuuuvv \rangle}{2} - \langle uuuuvuv \rangle - 2 \langle uuuuvvv \rangle - 3 \langle uuuvuvv \rangle + \frac{\langle uuuvvvv \rangle}{2}$$