

Pensieve header: Solving the WKO equations using free-Lie μ -calculus technology, running on sphere.

Solving the Equations

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SetDirectory["~/AcademicPensieve/2013-04"];
<< ./init.m
<< FreeLie.m
$SeriesShowDegree = 3; $SeriesCompareDegree = 5;
<< muCalculus-MathematicaV8.m

μCoefficients[expr_] := Flatten[Last[Reap[Collect[expr, _LW | _CW, Sow[#] &]]]];
μCoefficients[l_List] := μCoefficients /@ l;

α = MakeLieSeries[{"1", "2"}, αs];
β = MakeLieSeries[{"1", "2"}, βs];
γ = MakeCWSeries[{"1", "2"}, γs];
V = M[{1 → α, 2 → β}, γ];
κs[d_, 1] := If[OddQ[d], 0, κs[d]];
κ = MakeCWSeries[{"1"}, κs];
Unprotect[C]; C = M[{1 → MakeLieSeries[0]}, κ];

HardR4[d_, V_] :=
  Module[{lhs = R+[2, 3] ** R+[1, 3] ** V, rhs = V ** (R+[1, 3] // dΔ[1, 1, 2])},
    {lhs[1]@d - rhs[1]@d, lhs[2]@d - rhs[2]@d,
     lhs[3]@d - rhs[3]@d, lhs[W]@d - rhs[W]@d}
  ];

TwistEq[d_, V_] :=
  Module[{lhs = V ** θ[1, 2], rhs = R+[1, 2] ** (V // dσ[{1, 2} → {2, 1}])},
    {lhs[1]@d - rhs[1]@d, lhs[2]@d - rhs[2]@d, lhs[W]@d - rhs[W]@d}
  ];

UnitarityEq[d_, V_] :=
  Module[{lhs = V ** (V // dA[1] // dA[2]), rhs = dε[1] ∪ dε[2]},
    {lhs[1]@d - rhs[1]@d, lhs[2]@d - rhs[2]@d, lhs[W]@d - rhs[W]@d}
  ];

CapEq[d_, V_, C_] :=
  Module[{lhs = V ** (C // dΔ[1, 1, 2]) // dc[1] // dc[2], rhs = C ∪ (C // dσ[1, 2])},
    {lhs[W]@d - rhs[W]@d}
  ];

VerticalFlipEq[d_, V_] :=
  Module[{lhs = V ** (V // dS[1] // dS[2]), rhs = R+[1, 2]},
    {lhs[1]@d - rhs[1]@d, lhs[2]@d - rhs[2]@d, lhs[W]@d - rhs[W]@d}
  ];

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{V, C}

$$\left\{ M \left[\left\{ 1 \rightarrow \text{LS} \left[\overline{1} \alpha s[1, 1] + \overline{2} \alpha s[1, 2], \overline{12} \alpha s[2, 1], \overline{112} \alpha s[3, 1] + \overline{122} \alpha s[3, 2] \right], \right. \right. \\ \left. \left. 2 \rightarrow \text{LS} \left[\overline{1} \beta s[1, 1] + \overline{2} \beta s[1, 2], \overline{12} \beta s[2, 1], \overline{112} \beta s[3, 1] + \overline{122} \beta s[3, 2] \right] \right\}, \right. \\ \left. \text{CWS} \left[\overline{1} \gamma s[1, 1] + \overline{2} \gamma s[1, 2], \overline{11} \gamma s[2, 1] + \overline{12} \gamma s[2, 2] + \overline{22} \gamma s[2, 3], \right. \right. \\ \left. \left. \overline{111} \gamma s[3, 1] + \overline{112} \gamma s[3, 2] + \overline{122} \gamma s[3, 3] + \overline{222} \gamma s[3, 4] \right] \right\}, \\ M \left[\left\{ 1 \rightarrow \text{LS} [0, 0, 0] \right\}, \text{CWS} [0, \overline{11} \kappa s[2], 0] \right] \right\}$$

HardR4[2, V]

$$\left\{ -\langle 12 \rangle \alpha s[1, 1] \alpha s[1, 2] + \langle 12 \rangle \alpha s[1, 2] \beta s[1, 1], \right. \\ -\langle 12 \rangle \alpha s[1, 2] \beta s[1, 1] + \langle 12 \rangle \beta s[1, 1] \beta s[1, 2], \\ \left. -\frac{\langle 12 \rangle}{2} - \langle 12 \rangle \alpha s[1, 2] + \langle 12 \rangle \beta s[1, 1], \frac{1}{2} \text{CW}[12] \alpha s[1, 1] \alpha s[1, 2] - \text{CW}[12] \alpha s[2, 1] - \right. \\ \left. \text{CW}[12] \alpha s[1, 2] \beta s[1, 1] + \frac{1}{2} \text{CW}[12] \beta s[1, 1] \beta s[1, 2] + \text{CW}[12] \beta s[2, 1] \right\}$$

HardR4[2, V] // μCoefficients

$$\left\{ \{-\alpha s[1, 1] \alpha s[1, 2] + \alpha s[1, 2] \beta s[1, 1]\}, \right. \\ \left\{ -\alpha s[1, 2] \beta s[1, 1] + \beta s[1, 1] \beta s[1, 2] \right\}, \left\{ -\frac{1}{2} - \alpha s[1, 2] + \beta s[1, 1] \right\}, \\ \left. \left\{ \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] - \alpha s[2, 1] - \alpha s[1, 2] \beta s[1, 1] + \frac{1}{2} \beta s[1, 1] \beta s[1, 2] + \beta s[2, 1] \right\} \right\}$$

TwistEq[1, V] // μCoefficients

$$\left\{ \{\alpha s[1, 1] - \beta s[1, 1], \alpha s[1, 2] - \beta s[1, 2]\}, \right. \\ \left\{ -\frac{1}{2} - \alpha s[1, 1] + \beta s[1, 1], -\frac{1}{2} - \alpha s[1, 2] + \beta s[1, 2] \right\}, \\ \left. \left\{ -\frac{1}{2} - \beta s[1, 1] + \gamma s[1, 1] - \gamma s[1, 2], -1 - \alpha s[1, 2] - \gamma s[1, 1] + \gamma s[1, 2] \right\} \right\}$$

CapEq[1, V, C] // μCoefficients

$$\left\{ \{-\alpha s[1, 1] + \gamma s[1, 1], -\beta s[1, 2] + \gamma s[1, 2]\} \right\}$$

VerticalFlipEq[1, V]

$$\{0, -\langle 1 \rangle, -2 \text{CW}[1] \alpha s[1, 1] - 2 \text{CW}[2] \beta s[1, 2]\}$$

VerticalFlipEq[1, V] // μCoefficients

$$\left\{ \{0\}, \{-1\}, \{-2 \alpha s[1, 1], -2 \beta s[1, 2]\} \right\}$$

solve[eqns_List] :=

Solve[(# == 0) & /@ eqns, Union[Cases[eqns, _αs | _βs | _γs | _κs, Infinity]]];

```
sol1 = solve[Union[μCoefficients[{
  HardR4[2, V], TwistEq[1, V], CapEq[1, V, C], VerticalFlipEq[1, V]
}]]]

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{}
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sol1 /. Rule -> Set; {V, C}
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{M[{1 -> LS[1̄ αs[1, 1] + 2̄ αs[1, 2], 1̄2̄ αs[2, 1], 1̄1̄2̄ αs[3, 1] + 1̄2̄2̄ αs[3, 2]],
  2 -> LS[1̄ βs[1, 1] + 2̄ βs[1, 2], 1̄2̄ βs[2, 1], 1̄1̄2̄ βs[3, 1] + 1̄2̄2̄ βs[3, 2]]},
  CWS[1̄ γs[1, 1] + 2̄ γs[1, 2], 1̄1̄ γs[2, 1] + 1̄2̄ γs[2, 2] + 2̄2̄ γs[2, 3],
  1̄1̄1̄ γs[3, 1] + 1̄1̄2̄ γs[3, 2] + 1̄2̄2̄ γs[3, 3] + 2̄2̄2̄ γs[3, 4]]],
  M[{1 -> LS[0, 0, 0]}, CWS[0, 1̄1̄ κs[2], 0]]}]
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UnitarityEq[2, V]
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{-⟨12⟩ αs[1, 1] αs[1, 2] + ⟨12⟩ αs[1, 2] βs[1, 1],
 -⟨12⟩ αs[1, 2] βs[1, 1] + ⟨12⟩ βs[1, 1] βs[1, 2], 2 CW[12] αs[1, 1] αs[1, 2] +
  2 CW[12] αs[2, 1] - 4 CW[12] αs[1, 2] βs[1, 1] + 2 CW[12] βs[1, 1] βs[1, 2] -
  2 CW[12] βs[2, 1] + 2 CW[11] γs[2, 1] + 2 CW[12] γs[2, 2] + 2 CW[22] γs[2, 3]}
```

HardR4[3, V] // μ Coefficients

$$\begin{aligned}
& \left\{ \left\{ \frac{1}{2} \alpha s[1, 1]^2 \alpha s[1, 2] - \alpha s[1, 1] \alpha s[2, 1] - \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] + \right. \right. \\
& \quad \alpha s[2, 1] \beta s[1, 1] + \frac{1}{2} \alpha s[1, 2] \beta s[1, 1]^2, \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2]^2 - \alpha s[1, 2] \alpha s[2, 1] - \\
& \quad \left. \left. \alpha s[1, 2]^2 \beta s[1, 1] + \frac{1}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + \alpha s[1, 2] \beta s[2, 1] \right\}, \right. \\
& \left\{ \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] - \alpha s[2, 1] \beta s[1, 1] - \alpha s[1, 2] \beta s[1, 1]^2 + \right. \\
& \quad \left. \frac{1}{2} \beta s[1, 1]^2 \beta s[1, 2] + \beta s[1, 1] \beta s[2, 1], \right. \\
& \left. \frac{1}{2} \alpha s[1, 2]^2 \beta s[1, 1] - \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + \right. \\
& \quad \left. \frac{1}{2} \beta s[1, 1] \beta s[1, 2]^2 - \alpha s[1, 2] \beta s[2, 1] + \beta s[1, 2] \beta s[2, 1] \right\}, \\
& \left\{ \frac{1}{12} + \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] - \alpha s[2, 1] - \frac{1}{2} \beta s[1, 1] - \alpha s[1, 2] \beta s[1, 1] + \frac{1}{2} \beta s[1, 1]^2, \right. \\
& \quad \left. \frac{1}{12} + \frac{1}{2} \alpha s[1, 2] + \frac{1}{2} \alpha s[1, 2]^2 - \alpha s[1, 2] \beta s[1, 1] + \frac{1}{2} \beta s[1, 1] \beta s[1, 2] + \beta s[2, 1] \right\}, \\
& \left\{ -\frac{1}{6} \alpha s[1, 1]^2 \alpha s[1, 2] + \frac{1}{2} \alpha s[1, 1] \alpha s[2, 1] - \alpha s[3, 1] + \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] - \right. \\
& \quad \alpha s[2, 1] \beta s[1, 1] - \frac{1}{2} \alpha s[1, 2] \beta s[1, 1]^2 + \frac{1}{6} \beta s[1, 1]^2 \beta s[1, 2] + \\
& \quad \frac{1}{2} \beta s[1, 1] \beta s[2, 1] + \beta s[3, 1], \frac{1}{6} \alpha s[1, 1] \alpha s[1, 2]^2 - \frac{1}{2} \alpha s[1, 2] \alpha s[2, 1] + \\
& \quad \alpha s[3, 2] - \frac{1}{2} \alpha s[1, 2]^2 \beta s[1, 1] + \frac{1}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] - \\
& \quad \left. \left. \frac{1}{6} \beta s[1, 1] \beta s[1, 2]^2 + \alpha s[1, 2] \beta s[2, 1] - \frac{1}{2} \beta s[1, 2] \beta s[2, 1] - \beta s[3, 2] \right\} \right\}
\end{aligned}$$

TwistEq[2, V] // μ Coefficients

$$\begin{aligned}
& \left\{ \left\{ \frac{1}{2} \alpha s[1, 2] + \alpha s[2, 1] - \alpha s[1, 1] \beta s[1, 2] + \beta s[1, 1] \beta s[1, 2] - \beta s[2, 1] \right\}, \right. \\
& \left\{ -\frac{3}{8} - \frac{1}{2} \alpha s[1, 2] - \alpha s[1, 1] \alpha s[1, 2] - \alpha s[2, 1] - \right. \\
& \quad \left. \frac{1}{4} \beta s[1, 1] + \frac{5}{4} \beta s[1, 2] + \alpha s[1, 1] \beta s[1, 2] + \beta s[2, 1] \right\}, \\
& \left\{ -\frac{1}{4} - \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] - \alpha s[2, 1] + \alpha s[1, 1] \beta s[1, 2] - \frac{1}{2} \beta s[1, 1] \beta s[1, 2] + \beta s[2, 1], \right. \\
& \quad \left. \gamma s[2, 1] - \gamma s[2, 3], -\gamma s[2, 1] + \gamma s[2, 3] \right\} \left. \right\}
\end{aligned}$$

CapEq[2, V, C] // μCoefficients

$$\left\{ \left\{ \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] + \alpha s[2, 1] - \alpha s[1, 2] \beta s[1, 1] + \frac{1}{2} \beta s[1, 1] \beta s[1, 2] - \beta s[2, 1] + \gamma s[2, 2] + 2 \kappa s[2], \gamma s[2, 1], \gamma s[2, 3] \right\} \right\}$$

**sol2 = Solve[(# == 0) & /@ Union[μCoefficients[{
HardR4[3, V], TwistEq[2, V], UnitarityEq[2, V], CapEq[2, V, C]
}]]]**

{}

sol2 /. Rule -> Set; {V, C}

$$\left\{ M \left[\left\{ 1 \rightarrow LS \left[\overline{1} \alpha s[1, 1] + \overline{2} \alpha s[1, 2], \overline{12} \alpha s[2, 1], \overline{112} \alpha s[3, 1] + \overline{122} \alpha s[3, 2] \right], \right. \right. \\ \left. \left. 2 \rightarrow LS \left[\overline{1} \beta s[1, 1] + \overline{2} \beta s[1, 2], \overline{12} \beta s[2, 1], \overline{112} \beta s[3, 1] + \overline{122} \beta s[3, 2] \right] \right\}, \right. \\ \left. CWS \left[\overline{1} \gamma s[1, 1] + \overline{2} \gamma s[1, 2], \overline{11} \gamma s[2, 1] + \overline{12} \gamma s[2, 2] + \overline{22} \gamma s[2, 3], \right. \right. \\ \left. \left. \overline{111} \gamma s[3, 1] + \overline{112} \gamma s[3, 2] + \overline{122} \gamma s[3, 3] + \overline{222} \gamma s[3, 4] \right] \right\}, \\ M \left[\left\{ 1 \rightarrow LS[0, 0, 0] \right\}, CWS \left[0, \overline{11} \kappa s[2], 0 \right] \right\}$$

UnitarityEq[3, V] // μCoefficients

$$\left\{ \left\{ -\alpha s[1, 1]^2 \alpha s[1, 2] - \alpha s[1, 1] \alpha s[2, 1] + \frac{5}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] + \alpha s[2, 1] \beta s[1, 1] - \frac{3}{2} \alpha s[1, 2] \beta s[1, 1]^2, -\alpha s[1, 1] \alpha s[1, 2]^2 - \alpha s[1, 2] \alpha s[2, 1] + \frac{3}{2} \alpha s[1, 2]^2 \beta s[1, 1] - \frac{1}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + \alpha s[1, 2] \beta s[2, 1] \right\}, \right. \\ \left\{ -\frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] - \alpha s[2, 1] \beta s[1, 1] + \frac{3}{2} \alpha s[1, 2] \beta s[1, 1]^2 - \beta s[1, 1]^2 \beta s[1, 2] + \beta s[1, 1] \beta s[2, 1], \right. \\ \left. -\frac{3}{2} \alpha s[1, 2]^2 \beta s[1, 1] + \frac{5}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] - \beta s[1, 1] \beta s[1, 2]^2 - \alpha s[1, 2] \beta s[2, 1] + \beta s[1, 2] \beta s[2, 1] \right\}, \\ \left\{ \frac{4}{3} \alpha s[1, 1]^2 \alpha s[1, 2] + 2 \alpha s[1, 1] \alpha s[2, 1] + 2 \alpha s[3, 1] - 4 \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] - 4 \alpha s[2, 1] \beta s[1, 1] + 4 \alpha s[1, 2] \beta s[1, 1]^2 - \frac{4}{3} \beta s[1, 1]^2 \beta s[1, 2] + 2 \beta s[1, 1] \beta s[2, 1] - 2 \beta s[3, 1] + 2 \gamma s[3, 2], \right. \\ \left. -\frac{4}{3} \alpha s[1, 1] \alpha s[1, 2]^2 - 2 \alpha s[1, 2] \alpha s[2, 1] - 2 \alpha s[3, 2] + 4 \alpha s[1, 2]^2 \beta s[1, 1] - 4 \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + \frac{4}{3} \beta s[1, 1] \beta s[1, 2]^2 + 4 \alpha s[1, 2] \beta s[2, 1] - 2 \beta s[1, 2] \beta s[2, 1] + 2 \beta s[3, 2] + 2 \gamma s[3, 3], 2 \gamma s[3, 1], 2 \gamma s[3, 4] \right\} \right\}$$

HardR4[4, V] // μ Coefficients

$$\left\{ \left\{ -\frac{1}{6} \alpha s[1, 1]^3 \alpha s[1, 2] + \frac{1}{2} \alpha s[1, 1]^2 \alpha s[2, 1] - \alpha s[1, 1] \alpha s[3, 1] + \right. \right. \\
\frac{1}{2} \alpha s[1, 1]^2 \alpha s[1, 2] \beta s[1, 1] - \alpha s[1, 1] \alpha s[2, 1] \beta s[1, 1] + \alpha s[3, 1] \beta s[1, 1] - \\
\frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1]^2 + \frac{1}{2} \alpha s[2, 1] \beta s[1, 1]^2 + \frac{1}{6} \alpha s[1, 2] \beta s[1, 1]^3, \\
-\frac{2}{3} \alpha s[1, 1]^2 \alpha s[1, 2]^2 + 2 \alpha s[1, 1] \alpha s[1, 2] \alpha s[2, 1] - \alpha s[2, 1]^2 - \\
\alpha s[1, 2] \alpha s[3, 1] - \alpha s[1, 1] \alpha s[3, 2] + 2 \alpha s[1, 1] \alpha s[1, 2]^2 \beta s[1, 1] - \\
3 \alpha s[1, 2] \alpha s[2, 1] \beta s[1, 1] + \alpha s[3, 2] \beta s[1, 1] - \frac{3}{2} \alpha s[1, 2]^2 \beta s[1, 1]^2 - \\
\frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + \frac{1}{2} \alpha s[2, 1] \beta s[1, 1] \beta s[1, 2] + \\
\frac{2}{3} \alpha s[1, 2] \beta s[1, 1]^2 \beta s[1, 2] - \alpha s[1, 1] \alpha s[1, 2] \beta s[2, 1] + \\
\left. \alpha s[2, 1] \beta s[2, 1] + \frac{3}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[2, 1] + \alpha s[1, 2] \beta s[3, 1], \right. \\
-\frac{1}{6} \alpha s[1, 1] \alpha s[1, 2]^3 + \frac{1}{2} \alpha s[1, 2]^2 \alpha s[2, 1] - \alpha s[1, 2] \alpha s[3, 2] + \\
\frac{1}{2} \alpha s[1, 2]^3 \beta s[1, 1] - \frac{1}{2} \alpha s[1, 2]^2 \beta s[1, 1] \beta s[1, 2] + \frac{1}{6} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2]^2 - \\
\left. \alpha s[1, 2]^2 \beta s[2, 1] + \frac{1}{2} \alpha s[1, 2] \beta s[1, 2] \beta s[2, 1] + \alpha s[1, 2] \beta s[3, 2] \right\}, \\
\left\{ -\frac{1}{6} \alpha s[1, 1]^2 \alpha s[1, 2] \beta s[1, 1] + \frac{1}{2} \alpha s[1, 1] \alpha s[2, 1] \beta s[1, 1] - \alpha s[3, 1] \beta s[1, 1] + \right. \\
\frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1]^2 - \alpha s[2, 1] \beta s[1, 1]^2 - \frac{1}{2} \alpha s[1, 2] \beta s[1, 1]^3 + \\
\frac{1}{6} \beta s[1, 1]^3 \beta s[1, 2] + \frac{1}{2} \beta s[1, 1]^2 \beta s[2, 1] + \beta s[1, 1] \beta s[3, 1], \\
-\frac{2}{3} \alpha s[1, 1] \alpha s[1, 2]^2 \beta s[1, 1] + \frac{3}{2} \alpha s[1, 2] \alpha s[2, 1] \beta s[1, 1] - \alpha s[3, 2] \beta s[1, 1] + \\
\frac{3}{2} \alpha s[1, 2]^2 \beta s[1, 1]^2 + \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] - \\
\alpha s[2, 1] \beta s[1, 1] \beta s[1, 2] - 2 \alpha s[1, 2] \beta s[1, 1]^2 \beta s[1, 2] + \\
\frac{2}{3} \beta s[1, 1]^2 \beta s[1, 2]^2 + \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[2, 1] - \alpha s[2, 1] \beta s[2, 1] - \\
3 \alpha s[1, 2] \beta s[1, 1] \beta s[2, 1] + 2 \beta s[1, 1] \beta s[1, 2] \beta s[2, 1] + \\
\beta s[2, 1]^2 - \alpha s[1, 2] \beta s[3, 1] + \beta s[1, 2] \beta s[3, 1] + \beta s[1, 1] \beta s[3, 2], \\
-\frac{1}{6} \alpha s[1, 2]^3 \beta s[1, 1] + \frac{1}{2} \alpha s[1, 2]^2 \beta s[1, 1] \beta s[1, 2] - \frac{1}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2]^2 + \\
\frac{1}{6} \beta s[1, 1] \beta s[1, 2]^3 + \frac{1}{2} \alpha s[1, 2]^2 \beta s[2, 1] - \alpha s[1, 2] \beta s[1, 2] \beta s[2, 1] + \\
\left. \frac{1}{2} \beta s[1, 2]^2 \beta s[2, 1] - \alpha s[1, 2] \beta s[3, 2] + \beta s[1, 2] \beta s[3, 2] \right\},$$

$$\begin{aligned}
 & \left\{ -\frac{1}{24} - \frac{1}{12} \alpha s[1, 2] - \frac{1}{4} \alpha s[1, 1] \alpha s[1, 2] - \frac{2}{3} \alpha s[1, 1] \alpha s[1, 2]^2 + \frac{1}{2} \alpha s[2, 1] + \right. \\
 & \frac{3}{2} \alpha s[1, 2] \alpha s[2, 1] - \alpha s[3, 2] + \frac{1}{12} \beta s[1, 1] + \alpha s[1, 2] \beta s[1, 1] + \\
 & \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] + \frac{3}{2} \alpha s[1, 2]^2 \beta s[1, 1] - \alpha s[2, 1] \beta s[1, 1] - \\
 & \frac{3}{2} \alpha s[1, 2] \beta s[1, 1]^2 - \frac{1}{4} \beta s[1, 1] \beta s[1, 2] - \frac{1}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + \\
 & \frac{2}{3} \beta s[1, 1]^2 \beta s[1, 2] - \frac{1}{2} \beta s[2, 1] - \alpha s[1, 2] \beta s[2, 1] + \frac{3}{2} \beta s[1, 1] \beta s[2, 1] + \beta s[3, 1], \\
 & -\frac{1}{12} \alpha s[1, 2] - \frac{1}{4} \alpha s[1, 2]^2 - \frac{1}{6} \alpha s[1, 2]^3 + \frac{1}{2} \alpha s[1, 2]^2 \beta s[1, 1] - \\
 & \frac{1}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + \frac{1}{6} \beta s[1, 1] \beta s[1, 2]^2 - \alpha s[1, 2] \beta s[2, 1] + \\
 & \frac{1}{2} \beta s[1, 2] \beta s[2, 1] + \beta s[3, 2], -\frac{1}{6} \alpha s[1, 1]^2 \alpha s[1, 2] + \\
 & \frac{1}{2} \alpha s[1, 1] \alpha s[2, 1] - \alpha s[3, 1] + \frac{1}{12} \beta s[1, 1] + \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] - \\
 & \left. \alpha s[2, 1] \beta s[1, 1] - \frac{1}{4} \beta s[1, 1]^2 - \frac{1}{2} \alpha s[1, 2] \beta s[1, 1]^2 + \frac{1}{6} \beta s[1, 1]^3 \right\}, \\
 & \left\{ \frac{1}{24} \alpha s[1, 1]^3 \alpha s[1, 2] - \frac{1}{6} \alpha s[1, 1]^2 \alpha s[2, 1] + \frac{1}{2} \alpha s[1, 1] \alpha s[3, 1] - \alpha s[4, 1] - \right. \\
 & \frac{1}{6} \alpha s[1, 1]^2 \alpha s[1, 2] \beta s[1, 1] + \frac{1}{2} \alpha s[1, 1] \alpha s[2, 1] \beta s[1, 1] - \alpha s[3, 1] \beta s[1, 1] + \\
 & \frac{1}{4} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1]^2 - \frac{1}{2} \alpha s[2, 1] \beta s[1, 1]^2 - \frac{1}{6} \alpha s[1, 2] \beta s[1, 1]^3 + \\
 & \frac{1}{24} \beta s[1, 1]^3 \beta s[1, 2] + \frac{1}{6} \beta s[1, 1]^2 \beta s[2, 1] + \frac{1}{2} \beta s[1, 1] \beta s[3, 1] + \beta s[4, 1], \\
 & \frac{5}{12} \alpha s[1, 1]^2 \alpha s[1, 2]^2 - \frac{5}{3} \alpha s[1, 1] \alpha s[1, 2] \alpha s[2, 1] + \frac{3}{2} \alpha s[2, 1]^2 + \\
 & \frac{1}{2} \alpha s[1, 2] \alpha s[3, 1] + \frac{1}{2} \alpha s[1, 1] \alpha s[3, 2] - \alpha s[4, 2] - \frac{5}{3} \alpha s[1, 1] \alpha s[1, 2]^2 \beta s[1, 1] + \\
 & \frac{7}{2} \alpha s[1, 2] \alpha s[2, 1] \beta s[1, 1] - \alpha s[3, 2] \beta s[1, 1] + \frac{7}{4} \alpha s[1, 2]^2 \beta s[1, 1]^2 + \\
 & \frac{3}{4} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] - \frac{3}{2} \alpha s[2, 1] \beta s[1, 1] \beta s[1, 2] - \\
 & \frac{5}{3} \alpha s[1, 2] \beta s[1, 1]^2 \beta s[1, 2] + \frac{5}{12} \beta s[1, 1]^2 \beta s[1, 2]^2 + \frac{3}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[2, 1] - \\
 & 3 \alpha s[2, 1] \beta s[2, 1] - \frac{7}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[2, 1] + \frac{5}{3} \beta s[1, 1] \beta s[1, 2] \beta s[2, 1] + \\
 & \frac{3}{2} \beta s[2, 1]^2 - \alpha s[1, 2] \beta s[3, 1] + \frac{1}{2} \beta s[1, 2] \beta s[3, 1] + \frac{1}{2} \beta s[1, 1] \beta s[3, 2] + \beta s[4, 2], \\
 & -\frac{7}{12} \alpha s[1, 1]^2 \alpha s[1, 2]^2 + \frac{7}{3} \alpha s[1, 1] \alpha s[1, 2] \alpha s[2, 1] - 2 \alpha s[2, 1]^2 - \\
 & \left. \alpha s[1, 2] \alpha s[3, 1] - \alpha s[1, 1] \alpha s[3, 2] + 2 \alpha s[4, 2] + \frac{7}{3} \alpha s[1, 1] \alpha s[1, 2]^2 \beta s[1, 1] - \right.
 \end{aligned}$$

$$\begin{aligned}
& 5 \alpha s[1, 2] \alpha s[2, 1] \beta s[1, 1] + 2 \alpha s[3, 2] \beta s[1, 1] - \frac{5}{2} \alpha s[1, 2]^2 \beta s[1, 1]^2 - \\
& \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + 2 \alpha s[2, 1] \beta s[1, 1] \beta s[1, 2] + \\
& \frac{7}{3} \alpha s[1, 2] \beta s[1, 1]^2 \beta s[1, 2] - \frac{7}{12} \beta s[1, 1]^2 \beta s[1, 2]^2 - 2 \alpha s[1, 1] \alpha s[1, 2] \beta s[2, 1] + \\
& 4 \alpha s[2, 1] \beta s[2, 1] + 5 \alpha s[1, 2] \beta s[1, 1] \beta s[2, 1] - \frac{7}{3} \beta s[1, 1] \beta s[1, 2] \beta s[2, 1] - \\
& 2 \beta s[2, 1]^2 + 2 \alpha s[1, 2] \beta s[3, 1] - \beta s[1, 2] \beta s[3, 1] - \beta s[1, 1] \beta s[3, 2] - 2 \beta s[4, 2], \\
& \frac{1}{24} \alpha s[1, 1] \alpha s[1, 2]^3 - \frac{1}{6} \alpha s[1, 2]^2 \alpha s[2, 1] + \frac{1}{2} \alpha s[1, 2] \alpha s[3, 2] - \alpha s[4, 3] - \\
& \frac{1}{6} \alpha s[1, 2]^3 \beta s[1, 1] + \frac{1}{4} \alpha s[1, 2]^2 \beta s[1, 1] \beta s[1, 2] - \frac{1}{6} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2]^2 + \\
& \frac{1}{24} \beta s[1, 1] \beta s[1, 2]^3 + \frac{1}{2} \alpha s[1, 2]^2 \beta s[2, 1] - \frac{1}{2} \alpha s[1, 2] \beta s[1, 2] \beta s[2, 1] + \\
& \left. \frac{1}{6} \beta s[1, 2]^2 \beta s[2, 1] - \alpha s[1, 2] \beta s[3, 2] + \frac{1}{2} \beta s[1, 2] \beta s[3, 2] + \beta s[4, 3] \right\}
\end{aligned}$$

TwistEq[3, V] // μ Coefficients

$$\left\{ \left\{ \frac{1}{8} \alpha s[1, 2] + \frac{1}{2} \alpha s[2, 1] + \alpha s[3, 1] - \frac{1}{2} \alpha s[1, 1]^2 \beta s[1, 2] + \alpha s[1, 1] \beta s[1, 1] \beta s[1, 2] - \frac{1}{2} \beta s[1, 1]^2 \beta s[1, 2] - \alpha s[1, 1] \beta s[2, 1] + \beta s[1, 1] \beta s[2, 1] - \beta s[3, 1], \right. \right. \\
 \left. - \frac{1}{4} \alpha s[1, 2] + \alpha s[3, 2] - \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 2] - \alpha s[2, 1] \beta s[1, 2] + \alpha s[1, 1] \beta s[1, 2]^2 - \frac{1}{2} \beta s[1, 1] \beta s[1, 2]^2 + \beta s[1, 2] \beta s[2, 1] - \beta s[3, 2] \right\}, \\
 \left\{ -\frac{13}{96} - \frac{1}{12} \alpha s[1, 2] - \frac{5}{12} \alpha s[1, 1] \alpha s[1, 2] - \frac{1}{2} \alpha s[1, 1]^2 \alpha s[1, 2] - \frac{1}{2} \alpha s[2, 1] - \alpha s[1, 1] \alpha s[2, 1] - \alpha s[3, 1] - \frac{1}{6} \beta s[1, 1] - \frac{1}{24} \beta s[1, 1]^2 + \frac{1}{48} \beta s[1, 2] + \alpha s[1, 1] \beta s[1, 2] + \alpha s[1, 1]^2 \beta s[1, 2] - \frac{13}{24} \beta s[1, 1] \beta s[1, 2] - \frac{1}{2} \alpha s[1, 1] \beta s[1, 1] \beta s[1, 2] + \frac{5}{4} \beta s[2, 1] + \alpha s[1, 1] \beta s[2, 1] + \beta s[3, 1], \right. \\
 \left. \frac{13}{96} - \frac{1}{12} \alpha s[1, 2]^2 - \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2]^2 - \alpha s[1, 2] \alpha s[2, 1] - \alpha s[3, 2] + \frac{1}{48} \beta s[1, 1] - \frac{1}{6} \beta s[1, 2] + \frac{1}{2} \alpha s[1, 2] \beta s[1, 2] + \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 2] + \alpha s[2, 1] \beta s[1, 2] + \frac{1}{24} \beta s[1, 1] \beta s[1, 2] - \frac{11}{24} \beta s[1, 2]^2 - \frac{1}{2} \alpha s[1, 1] \beta s[1, 2]^2 - \frac{1}{4} \beta s[2, 1] + \beta s[3, 2] \right\}, \\
 \left\{ -\frac{1}{16} - \frac{1}{6} \alpha s[1, 1]^2 \alpha s[1, 2] - \frac{1}{2} \alpha s[1, 1] \alpha s[2, 1] - \alpha s[3, 1] + \frac{1}{2} \alpha s[1, 1]^2 \beta s[1, 2] - \frac{1}{2} \alpha s[1, 1] \beta s[1, 1] \beta s[1, 2] + \frac{1}{6} \beta s[1, 1]^2 \beta s[1, 2] + \alpha s[1, 1] \beta s[2, 1] - \frac{1}{2} \beta s[1, 1] \beta s[2, 1] + \beta s[3, 1] + \gamma s[3, 2] - \gamma s[3, 3], \right. \\
 \left. -\frac{1}{16} + \frac{1}{6} \alpha s[1, 1] \alpha s[1, 2]^2 + \frac{1}{2} \alpha s[1, 2] \alpha s[2, 1] + \alpha s[3, 2] - \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 2] - \alpha s[2, 1] \beta s[1, 2] + \frac{1}{2} \alpha s[1, 1] \beta s[1, 2]^2 - \frac{1}{6} \beta s[1, 1] \beta s[1, 2]^2 + \frac{1}{2} \beta s[1, 2] \beta s[2, 1] - \beta s[3, 2] - \gamma s[3, 2] + \gamma s[3, 3], \right. \\
 \left. \gamma s[3, 1] - \gamma s[3, 4], -\gamma s[3, 1] + \gamma s[3, 4] \right\} \left. \right\}$$

CapEq[3, V, C] // μCoefficients

$$\left\{ \left\{ \frac{1}{6} \alpha s[1, 1]^2 \alpha s[1, 2] + \frac{1}{2} \alpha s[1, 1] \alpha s[2, 1] + \alpha s[3, 1] - \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] - \alpha s[2, 1] \beta s[1, 1] + \frac{1}{2} \alpha s[1, 2] \beta s[1, 1]^2 - \frac{1}{6} \beta s[1, 1]^2 \beta s[1, 2] + \frac{1}{2} \beta s[1, 1] \beta s[2, 1] - \beta s[3, 1] + \gamma s[3, 2], -\frac{1}{6} \alpha s[1, 1] \alpha s[1, 2]^2 - \frac{1}{2} \alpha s[1, 2] \alpha s[2, 1] - \alpha s[3, 2] + \frac{1}{2} \alpha s[1, 2]^2 \beta s[1, 1] - \frac{1}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + \frac{1}{6} \beta s[1, 1] \beta s[1, 2]^2 + \alpha s[1, 2] \beta s[2, 1] - \frac{1}{2} \beta s[1, 2] \beta s[2, 1] + \beta s[3, 2] + \gamma s[3, 3], \gamma s[3, 1], \gamma s[3, 4] \right\} \right\}$$

sol3 = Solve[(# == 0) & /@ Union[μCoefficients[{ HardR4[4, V], TwistEq[3, V], UnitarityEq[3, V], CapEq[3, V, C] }]]]]

{}

sol3 /. Rule -> Set; {V, C}

$$\left\{ M \left[\left\{ 1 \rightarrow LS \left[\overline{1} \alpha s[1, 1] + \overline{2} \alpha s[1, 2], \overline{12} \alpha s[2, 1], \overline{112} \alpha s[3, 1] + \overline{122} \alpha s[3, 2] \right], 2 \rightarrow LS \left[\overline{1} \beta s[1, 1] + \overline{2} \beta s[1, 2], \overline{12} \beta s[2, 1], \overline{112} \beta s[3, 1] + \overline{122} \beta s[3, 2] \right] \right\}, CWS \left[\overline{1} \gamma s[1, 1] + \overline{2} \gamma s[1, 2], \overline{11} \gamma s[2, 1] + \overline{12} \gamma s[2, 2] + \overline{22} \gamma s[2, 3], \overline{111} \gamma s[3, 1] + \overline{112} \gamma s[3, 2] + \overline{122} \gamma s[3, 3] + \overline{222} \gamma s[3, 4] \right] \right], M \left[\left\{ 1 \rightarrow LS[0, 0, 0] \right\}, CWS \left[0, \overline{11} \kappa s[2], 0 \right] \right] \right\}$$

UnitarityEq[4, V] // μCoefficients

$$\left\{ \left\{ -\frac{7}{12} \alpha s[1, 1]^3 \alpha s[1, 2] - \alpha s[1, 1]^2 \alpha s[2, 1] - \alpha s[1, 1] \alpha s[3, 1] + \frac{13}{6} \alpha s[1, 1]^2 \alpha s[1, 2] \beta s[1, 1] + \frac{5}{2} \alpha s[1, 1] \alpha s[2, 1] \beta s[1, 1] + \alpha s[3, 1] \beta s[1, 1] - \frac{11}{4} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1]^2 - \frac{3}{2} \alpha s[2, 1] \beta s[1, 1]^2 + \frac{7}{6} \alpha s[1, 2] \beta s[1, 1]^3, -\frac{17}{6} \alpha s[1, 1]^2 \alpha s[1, 2]^2 - 4 \alpha s[1, 1] \alpha s[1, 2] \alpha s[2, 1] - \alpha s[2, 1]^2 - \alpha s[1, 2] \alpha s[3, 1] - \alpha s[1, 1] \alpha s[3, 2] + \frac{26}{3} \alpha s[1, 1] \alpha s[1, 2]^2 \beta s[1, 1] + \frac{11}{2} \alpha s[1, 2] \alpha s[2, 1] \beta s[1, 1] + \alpha s[3, 2] \beta s[1, 1] - \frac{25}{4} \alpha s[1, 2]^2 \beta s[1, 1]^2 - \frac{9}{4} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] - \frac{1}{2} \alpha s[2, 1] \beta s[1, 1] \beta s[1, 2] + \frac{8}{3} \alpha s[1, 2] \beta s[1, 1]^2 \beta s[1, 2] + \frac{5}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[2, 1] + \right. \right\}$$

$$\begin{aligned}
 & \alpha s[2, 1] \beta s[2, 1] - \frac{7}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[2, 1] + \alpha s[1, 2] \beta s[3, 1], \\
 & - \frac{7}{12} \alpha s[1, 1] \alpha s[1, 2]^3 - \alpha s[1, 2]^2 \alpha s[2, 1] - \alpha s[1, 2] \alpha s[3, 2] + \\
 & \frac{7}{6} \alpha s[1, 2]^3 \beta s[1, 1] - \frac{3}{4} \alpha s[1, 2]^2 \beta s[1, 1] \beta s[1, 2] + \frac{1}{6} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2]^2 + \\
 & \left. \frac{3}{2} \alpha s[1, 2]^2 \beta s[2, 1] - \frac{1}{2} \alpha s[1, 2] \beta s[1, 2] \beta s[2, 1] + \alpha s[1, 2] \beta s[3, 2] \right\}, \\
 & \left\{ - \frac{1}{6} \alpha s[1, 1]^2 \alpha s[1, 2] \beta s[1, 1] - \frac{1}{2} \alpha s[1, 1] \alpha s[2, 1] \beta s[1, 1] - \alpha s[3, 1] \beta s[1, 1] + \right. \\
 & \frac{3}{4} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1]^2 + \frac{3}{2} \alpha s[2, 1] \beta s[1, 1]^2 - \frac{7}{6} \alpha s[1, 2] \beta s[1, 1]^3 + \\
 & \frac{7}{12} \beta s[1, 1]^3 \beta s[1, 2] - \beta s[1, 1]^2 \beta s[2, 1] + \beta s[1, 1] \beta s[3, 1], \\
 & - \frac{8}{3} \alpha s[1, 1] \alpha s[1, 2]^2 \beta s[1, 1] - \frac{7}{2} \alpha s[1, 2] \alpha s[2, 1] \beta s[1, 1] - \alpha s[3, 2] \beta s[1, 1] + \\
 & \frac{25}{4} \alpha s[1, 2]^2 \beta s[1, 1]^2 + \frac{9}{4} \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + \\
 & \frac{5}{2} \alpha s[2, 1] \beta s[1, 1] \beta s[1, 2] - \frac{26}{3} \alpha s[1, 2] \beta s[1, 1]^2 \beta s[1, 2] + \\
 & \frac{17}{6} \beta s[1, 1]^2 \beta s[1, 2]^2 - \frac{1}{2} \alpha s[1, 1] \alpha s[1, 2] \beta s[2, 1] - \alpha s[2, 1] \beta s[2, 1] + \\
 & \frac{11}{2} \alpha s[1, 2] \beta s[1, 1] \beta s[2, 1] - 4 \beta s[1, 1] \beta s[1, 2] \beta s[2, 1] + \\
 & \beta s[2, 1]^2 - \alpha s[1, 2] \beta s[3, 1] + \beta s[1, 2] \beta s[3, 1] + \beta s[1, 1] \beta s[3, 2], \\
 & - \frac{7}{6} \alpha s[1, 2]^3 \beta s[1, 1] + \frac{11}{4} \alpha s[1, 2]^2 \beta s[1, 1] \beta s[1, 2] - \frac{13}{6} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2]^2 + \\
 & \left. \frac{7}{12} \beta s[1, 1] \beta s[1, 2]^3 - \frac{3}{2} \alpha s[1, 2]^2 \beta s[2, 1] + \frac{5}{2} \alpha s[1, 2] \beta s[1, 2] \beta s[2, 1] - \right. \\
 & \left. \beta s[1, 2]^2 \beta s[2, 1] - \alpha s[1, 2] \beta s[3, 2] + \beta s[1, 2] \beta s[3, 2] \right\}, \\
 & \left\{ \frac{2}{3} \alpha s[1, 1]^3 \alpha s[1, 2] + \frac{4}{3} \alpha s[1, 1]^2 \alpha s[2, 1] + 2 \alpha s[1, 1] \alpha s[3, 1] + \right. \\
 & 2 \alpha s[4, 1] - \frac{8}{3} \alpha s[1, 1]^2 \alpha s[1, 2] \beta s[1, 1] - 4 \alpha s[1, 1] \alpha s[2, 1] \beta s[1, 1] - \\
 & 4 \alpha s[3, 1] \beta s[1, 1] + 4 \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1]^2 + \\
 & 4 \alpha s[2, 1] \beta s[1, 1]^2 - \frac{8}{3} \alpha s[1, 2] \beta s[1, 1]^3 + \frac{2}{3} \beta s[1, 1]^3 \beta s[1, 2] - \\
 & \frac{4}{3} \beta s[1, 1]^2 \beta s[2, 1] + 2 \beta s[1, 1] \beta s[3, 1] - 2 \beta s[4, 1] + 2 \gamma s[4, 2], \\
 & \frac{31}{6} \alpha s[1, 1]^2 \alpha s[1, 2]^2 + \frac{28}{3} \alpha s[1, 1] \alpha s[1, 2] \alpha s[2, 1] + 4 \alpha s[2, 1]^2 + \\
 & 2 \alpha s[1, 2] \alpha s[3, 1] + 2 \alpha s[1, 1] \alpha s[3, 2] + 2 \alpha s[4, 2] - \frac{62}{3} \alpha s[1, 1] \alpha s[1, 2]^2 \beta s[1, 1] - \\
 & 20 \alpha s[1, 2] \alpha s[2, 1] \beta s[1, 1] - 4 \alpha s[3, 2] \beta s[1, 1] + 22 \alpha s[1, 2]^2 \beta s[1, 1]^2 + \\
 & \left. 9 \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] + 8 \alpha s[2, 1] \beta s[1, 1] \beta s[1, 2] - \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{62}{3} \alpha s[1, 2] \beta s[1, 1]^2 \beta s[1, 2] + \frac{31}{6} \beta s[1, 1]^2 \beta s[1, 2]^2 - 8 \alpha s[1, 1] \alpha s[1, 2] \beta s[2, 1] - \\
 & 8 \alpha s[2, 1] \beta s[2, 1] + 20 \alpha s[1, 2] \beta s[1, 1] \beta s[2, 1] - \frac{28}{3} \beta s[1, 1] \beta s[1, 2] \beta s[2, 1] + \\
 & 4 \beta s[2, 1]^2 - 4 \alpha s[1, 2] \beta s[3, 1] + 2 \beta s[1, 2] \beta s[3, 1] + 2 \beta s[1, 1] \beta s[3, 2] - 2 \beta s[4, 2] + \\
 & 4 \alpha s[1, 1] \alpha s[1, 2] \gamma s[2, 2] + 4 \alpha s[2, 1] \gamma s[2, 2] - 8 \alpha s[1, 2] \beta s[1, 1] \gamma s[2, 2] + \\
 & 4 \beta s[1, 1] \beta s[1, 2] \gamma s[2, 2] - 4 \beta s[2, 1] \gamma s[2, 2] + 2 \gamma s[4, 3], \\
 & - \frac{47}{6} \alpha s[1, 1]^2 \alpha s[1, 2]^2 - \frac{44}{3} \alpha s[1, 1] \alpha s[1, 2] \alpha s[2, 1] - 6 \alpha s[2, 1]^2 - \\
 & 4 \alpha s[1, 2] \alpha s[3, 1] - 4 \alpha s[1, 1] \alpha s[3, 2] - 4 \alpha s[4, 2] + \frac{94}{3} \alpha s[1, 1] \alpha s[1, 2]^2 \beta s[1, 1] + \\
 & 32 \alpha s[1, 2] \alpha s[2, 1] \beta s[1, 1] + 8 \alpha s[3, 2] \beta s[1, 1] - 34 \alpha s[1, 2]^2 \beta s[1, 1]^2 - \\
 & 13 \alpha s[1, 1] \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2] - 12 \alpha s[2, 1] \beta s[1, 1] \beta s[1, 2] + \\
 & \frac{94}{3} \alpha s[1, 2] \beta s[1, 1]^2 \beta s[1, 2] - \frac{47}{6} \beta s[1, 1]^2 \beta s[1, 2]^2 + 12 \alpha s[1, 1] \alpha s[1, 2] \beta s[2, 1] + \\
 & 12 \alpha s[2, 1] \beta s[2, 1] - 32 \alpha s[1, 2] \beta s[1, 1] \beta s[2, 1] + \frac{44}{3} \beta s[1, 1] \beta s[1, 2] \beta s[2, 1] - \\
 & 6 \beta s[2, 1]^2 + 8 \alpha s[1, 2] \beta s[3, 1] - 4 \beta s[1, 2] \beta s[3, 1] - 4 \beta s[1, 1] \beta s[3, 2] + 4 \beta s[4, 2] - \\
 & 4 \alpha s[1, 1] \alpha s[1, 2] \gamma s[2, 2] - 4 \alpha s[2, 1] \gamma s[2, 2] + 8 \alpha s[1, 2] \beta s[1, 1] \gamma s[2, 2] - \\
 & 4 \beta s[1, 1] \beta s[1, 2] \gamma s[2, 2] + 4 \beta s[2, 1] \gamma s[2, 2] + 2 \gamma s[4, 4], \\
 & \frac{2}{3} \alpha s[1, 1] \alpha s[1, 2]^3 + \frac{4}{3} \alpha s[1, 2]^2 \alpha s[2, 1] + 2 \alpha s[1, 2] \alpha s[3, 2] + \\
 & 2 \alpha s[4, 3] - \frac{8}{3} \alpha s[1, 2]^3 \beta s[1, 1] + 4 \alpha s[1, 2]^2 \beta s[1, 1] \beta s[1, 2] - \\
 & \frac{8}{3} \alpha s[1, 2] \beta s[1, 1] \beta s[1, 2]^2 + \frac{2}{3} \beta s[1, 1] \beta s[1, 2]^3 - 4 \alpha s[1, 2]^2 \beta s[2, 1] + \\
 & 4 \alpha s[1, 2] \beta s[1, 2] \beta s[2, 1] - \frac{4}{3} \beta s[1, 2]^2 \beta s[2, 1] - 4 \alpha s[1, 2] \beta s[3, 2] + \\
 & 2 \beta s[1, 2] \beta s[3, 2] - 2 \beta s[4, 3] + 2 \gamma s[4, 5], 2 \gamma s[4, 1], 2 \gamma s[4, 6] \} \}
 \end{aligned}$$

```

sol4 = Solve[ (# == 0) & /@ Union[μCoefficients[{
    HardR4[5, V], TwistEq[4, V], UnitarityEq[4, V], CapEq[4, V, C]
}]]]

```

```

sol4 /. Rule -> Set; $SeriesShowDegree = 4; {V, C}

```

```

VerticalFlipEq[#, V] & /@ {2, 3, 4}

```

```

sol5 = Solve[ (# == 0) & /@ Union[μCoefficients[{
    HardR4[6, V], TwistEq[5, V], UnitarityEq[5, V], CapEq[5, V, C]
}]]]

```

```

sol5 /. Rule -> Set; $SeriesShowDegree = 5; {V, C}

```

```

VerticalFlipEq[5, V]

```

```

sol6 = Solve[ (# == 0) & /@ Union[μCoefficients[{
    HardR4[7, V], TwistEq[6, V], UnitarityEq[6, V], CapEq[6, V, C]
}]]]

```

```

sol6 /. Rule -> Set; $SeriesShowDegree = 6; {V, C}
VerticalFlipEq[6, V]

sol7 = Solve[ (# == 0) & /@ Union[ $\mu$ Coefficients[{
    HardR4[8, V], TwistEq[7, V], UnitarityEq[7, V], CapEq[7, V, C]
}]]]

sol7 /. Rule -> Set; $SeriesShowDegree = 7; {V, C}
VerticalFlipEq[7, V]

sol8 = Solve[ (# == 0) & /@ Union[ $\mu$ Coefficients[{
    HardR4[9, V], TwistEq[8, V], UnitarityEq[8, V], CapEq[8, V, C]
}]]]

 $\beta$ s[8, 27] = 0; sol8 /. Rule -> Set; $SeriesShowDegree = 8; {V, C}
VerticalFlipEq[8, V]

sol9 = Solve[ (# == 0) & /@ Union[ $\mu$ Coefficients[{
    HardR4[10, V], TwistEq[9, V], UnitarityEq[9, V], CapEq[9, V, C]
}]]]

sol9 /. Rule -> Set; $SeriesShowDegree = 9; {V, C}
VerticalFlipEq[9, V]

sol10 = Solve[ (# == 0) & /@ Union[ $\mu$ Coefficients[{
    HardR4[11, V], TwistEq[10, V], UnitarityEq[10, V], CapEq[10, V, C]
}]]]

sol10 /. Rule -> Set; $SeriesShowDegree = 10; {V, C}
VerticalFlipEq[10, V]

```