Cheat Sheet J - Verification

Pensieve header: Cheat Sheet J Verification; continues pensieve://2013-03/; continued pensieve://2013-12/.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-04"];
<< FreeLie.m;
tm[u_, v_, w_] := LieMorphism[⟨u⟩ → ⟨w⟩, ⟨v⟩ → ⟨w⟩];
CC[u_, γ_LieSeries] := LieMorphism[u → Ad[γ][u]];
CC_u_[γ_] := CC[u, γ];
RC[u_, γ_LieSeries, ub_][ser_] :=
  StableApply[LieMorphism[⟨u⟩ → Ad[γ][⟨ub⟩]], ser];
RC[u_, γ_LieSeries][ser_] := ser // RC[u, γ, ⟨υ⟩] // LieMorphism[⟨υ⟩ → ⟨u⟩];
RC_u_[γ_] := RC[u, γ];
Print /@ {{t = ⟨"t"⟩, u = ⟨"u"⟩, v = ⟨"v"⟩, w = ⟨"w"⟩},
    α = RandomLieSeries[{t, u, v}],
    β = RandomLieSeries[{t, u, v}],
    γ = RandomLieSeries[{t, u, v}]
  };
$SeriesShowDegree = 3; $SeriesCompareDegree = 6;
```

```
{⟨t⟩, ⟨u⟩, ⟨v⟩, ⟨w⟩}
```

$$\text{LS}\left[t-u,\ -\frac{\overline{tv}}{2}+\frac{\overline{uv}}{2},\ \frac{2}{3}\overline{t\overline{tu}}+\frac{11}{6}\overline{t\overline{tv}}-\frac{3}{2}\overline{t\overline{uv}}-\frac{7}{6}\overline{u\overline{uv}}-\frac{2}{3}\overline{\overline{tu}u}-\frac{2}{3}\overline{\overline{tv}u}+\frac{11}{6}\overline{\overline{tv}v}+\frac{2}{3}\overline{\overline{uv}v}\right]$$

$$\text{LS}\left[t+u-v,\ \frac{3\overline{tu}}{2}+\frac{\overline{uv}}{2},\ \frac{5}{6}\overline{t\overline{tu}}-\frac{5}{3}\overline{t\overline{tv}}-\frac{7}{6}\overline{t\overline{uv}}-\frac{4}{3}\overline{u\overline{uv}}-\overline{\overline{tu}u}-\frac{3}{2}\overline{\overline{tv}u}-\frac{1}{2}\overline{\overline{tv}v}+2\overline{\overline{uv}v}\right]$$

$$\text{LS}\left[t+2u+2v,\ -\frac{3\overline{tu}}{2}-\frac{\overline{tv}}{2}-\frac{3\overline{uv}}{2},\ \frac{5}{3}\overline{t\overline{tu}}-\overline{t\overline{tv}}-\frac{7}{6}\overline{t\overline{uv}}+\frac{5}{3}\overline{u\overline{uv}}-\frac{2}{3}\overline{\overline{tu}u}-\frac{2}{3}\overline{\overline{tv}u}+\frac{2}{3}\overline{\overline{tv}v}\right]$$

```
ad[u_, γ_LieSeries] := LieDerivation[u → b[γ, u]];
ad_u_[γ_] := ad[u, γ];
ϵ /: ϵ^2 = 0;
```

- Some preliminary testing

```
(α // RC_u[γ] // CC_u[-γ]) ≡ α
```

```
True
```

```
(α // RC_u[γ] // RC_u[-γ // RC_u[γ]]) ≡ α
```

```
True
```

1. The Definition of J

```
J[u_, γ_] := Module[{s}, ∫_0^1 (γ // RC_u[s γ] // div_u // CC_u[-s γ]) ⅆs];
J_u_[γ_] := J[u, γ];
```

```
Ju[α][{4}]
```

$$\mathrm{CWS}\Big[2\,\mathrm{CW}[u],\ -\mathrm{CW}[tu]+\frac{3\,\mathrm{CW}[uv]}{2},\ \frac{\mathrm{CW}[ttu]}{3}-\frac{\mathrm{CW}[tuu]}{3}-\frac{7\,\mathrm{CW}[tuv]}{6}+\frac{17\,\mathrm{CW}[tvu]}{12}-\frac{\mathrm{CW}[uuv]}{3}-\frac{5\,\mathrm{CW}[uvv]}{4},\ \frac{5\,\mathrm{CW}[tttu]}{4}+\frac{25\,\mathrm{CW}[ttuu]}{24}-\frac{31\,\mathrm{CW}[ttuv]}{12}-4\,\mathrm{CW}[ttvu]-\frac{37\,\mathrm{CW}[tutu]}{12}+\frac{67\,\mathrm{CW}[tutv]}{12}+\frac{5\,\mathrm{CW}[tuuu]}{4}-\frac{2\,\mathrm{CW}[tuuv]}{3}-\frac{10\,\mathrm{CW}[tuvu]}{3}-\frac{17\,\mathrm{CW}[tuvv]}{24}+\frac{23\,\mathrm{CW}[tvuu]}{6}+\frac{19\,\mathrm{CW}[tvuv]}{12}+\frac{7\,\mathrm{CW}[tvvu]}{4}+\frac{41\,\mathrm{CW}[uuuv]}{24}-\frac{5\,\mathrm{CW}[uuvv]}{4}+\frac{\mathrm{CW}[uvuv]}{4}+\frac{5\,\mathrm{CW}[uvvv]}{24}\Big]$$

2. The J_{uv} equation

```
Print /@ {
    0 → {α, β},
    1 → (t1 = Ju[α] + (Jv[β // RCu[α]] // CCu[-α])),
    2 → (t2 = Jv[β] + (Ju[α // RCv[β]] // CCv[-β])),
    3 → t1 ≡ t2
  };
```

$$0 \to \Big\{\mathrm{LS}\Big[t-u,\ -\frac{\overline{tv}}{2}+\frac{\overline{uv}}{2},\ \frac{2}{3}\,\overline{t\,\overline{tu}}+\frac{11}{6}\,\overline{t\,\overline{tv}}-\frac{3}{2}\,\overline{t\,\overline{uv}}-\frac{7}{6}\,\overline{u\,\overline{uv}}-\frac{2}{3}\,\overline{\overline{tu}\,u}-\frac{2}{3}\,\overline{\overline{tv}\,u}+\frac{11}{6}\,\overline{\overline{tv}\,v}+\frac{2}{3}\,\overline{\overline{uv}\,v}\Big],\ \mathrm{LS}\Big[t+u-v,\ \frac{3\,\overline{tu}}{2}+\frac{\overline{uv}}{2},\ \frac{5}{6}\,\overline{t\,\overline{tu}}-\frac{5}{3}\,\overline{t\,\overline{tv}}-\frac{7}{6}\,\overline{t\,\overline{uv}}-\frac{4}{3}\,\overline{u\,\overline{uv}}-\overline{\overline{tu}\,u}-\frac{3}{2}\,\overline{\overline{tv}\,u}-\frac{1}{2}\,\overline{\overline{tv}\,v}+2\,\overline{\overline{uv}\,v}\Big]\Big\}$$

$$1 \to \mathrm{CWS}\Big[-\mathrm{CW}[u]-\mathrm{CW}[v],\ -\frac{\mathrm{CW}[tu]}{2}-\frac{\mathrm{CW}[tv]}{2}-\frac{\mathrm{CW}[uv]}{2},\ \frac{\mathrm{CW}[ttu]}{2}-\frac{11\,\mathrm{CW}[ttv]}{6}+\frac{\mathrm{CW}[tuu]}{2}-\frac{17\,\mathrm{CW}[tuv]}{12}+\frac{41\,\mathrm{CW}[tvu]}{12}+\frac{\mathrm{CW}[tvv]}{3}-\frac{5\,\mathrm{CW}[uuv]}{6}-\frac{5\,\mathrm{CW}[uvv]}{4}\Big]$$

$$2 \to \mathrm{CWS}\Big[-\mathrm{CW}[u]-\mathrm{CW}[v],\ -\frac{\mathrm{CW}[tu]}{2}-\frac{\mathrm{CW}[tv]}{2}-\frac{\mathrm{CW}[uv]}{2},\ \frac{\mathrm{CW}[ttu]}{2}-\frac{11\,\mathrm{CW}[ttv]}{6}+\frac{\mathrm{CW}[tuu]}{2}-\frac{17\,\mathrm{CW}[tuv]}{12}+\frac{41\,\mathrm{CW}[tvu]}{12}+\frac{\mathrm{CW}[tvv]}{3}-\frac{5\,\mathrm{CW}[uuv]}{6}-\frac{5\,\mathrm{CW}[uvv]}{4}\Big]$$

$$3 \to \mathrm{True}$$

```
Print /@ {
    0 → {α, β},
    1 → (t1 = Ju[α] + (Jv[β // RCu[s α]] // CCu[-s α])),
    2 → (t2 = Jv[β] + (Ju[α // RCv[s β]] // CCv[-s β])),
    3 → t1 ≡ t2
  };
```

$$0 \to \left\{ LS\left[t-u, -\frac{\overline{tv}}{2}+\frac{\overline{uv}}{2}, \frac{2}{3}\overline{t\,\overline{tu}}+\frac{11}{6}\overline{t\,\overline{tv}}-\frac{3}{2}\overline{t\,\overline{uv}}-\frac{7}{6}\overline{u\,\overline{uv}}-\frac{2}{3}\overline{\overline{tu}\,u}-\frac{2}{3}\overline{\overline{tv}\,u}+\frac{11}{6}\overline{\overline{tv}\,v}+\frac{2}{3}\overline{\overline{uv}\,v}\right],\right.$$
$$\left. LS\left[t+u-v, \frac{3\overline{tu}}{2}+\frac{\overline{uv}}{2}, \frac{5}{6}\overline{t\,\overline{tu}}-\frac{5}{3}\overline{t\,\overline{tv}}-\frac{7}{6}\overline{t\,\overline{uv}}-\frac{4}{3}\overline{u\,\overline{uv}}-\overline{\overline{tu}\,u}-\frac{3}{2}\overline{\overline{tv}\,u}-\frac{1}{2}\overline{\overline{tv}\,v}+2\overline{\overline{uv}\,v}\right]\right\}$$

$$1 \to CWS\left[-CW[u]-CW[v], -\frac{CW[tu]}{2}-\frac{CW[tv]}{2}-\frac{CW[uv]}{2}, \frac{CW[ttu]}{2}-\frac{11\,CW[ttv]}{6}+\frac{CW[tuu]}{2}-\right.$$
$$\left.\frac{17\,CW[tuv]}{12}+\frac{35\,CW[tvu]}{12}+\frac{1}{2}s\,CW[tvu]+\frac{CW[tvv]}{3}-\frac{CW[uuv]}{3}-\frac{1}{2}s\,CW[uuv]-\frac{5\,CW[uvv]}{4}\right]$$

$$2 \to CWS\left[-CW[u]-CW[v], -\frac{CW[tu]}{2}-\frac{CW[tv]}{2}-\frac{CW[uv]}{2}, \frac{CW[ttu]}{2}-\frac{11\,CW[ttv]}{6}+\frac{CW[tuu]}{2}-\right.$$
$$\left.\frac{17\,CW[tuv]}{12}+\frac{35\,CW[tvu]}{12}+\frac{1}{2}s\,CW[tvu]+\frac{CW[tvv]}{3}-\frac{CW[uuv]}{3}-\frac{1}{2}s\,CW[uuv]-\frac{5\,CW[uvv]}{4}\right]$$

$$3 \to \frac{25\,CW[tttu]}{24}-\frac{13\,CW[tttv]}{6}+\frac{11\,CW[ttuu]}{12}+\frac{5\,CW[ttuv]}{24}-\frac{11\,CW[ttvu]}{24}-\frac{11}{6}s\,CW[ttvu]+$$
$$\frac{1}{4}s^2\,CW[ttvu]+\frac{29\,CW[ttvv]}{24}-\frac{25\,CW[tutu]}{12}+\frac{5\,CW[tutv]}{12}+s\,CW[tutv]-\frac{11\,CW[tuuu]}{12}+$$
$$\frac{2\,CW[tuuv]}{3}-s\,CW[tuuv]-\frac{17\,CW[tuvu]}{4}+\frac{3}{2}s\,CW[tuvu]-\frac{1}{4}s^2\,CW[tuvu]+\frac{7\,CW[tuvv]}{4}-$$
$$\frac{8\,CW[tvtv]}{3}+\frac{11\,CW[tvuu]}{8}-\frac{5}{12}s\,CW[tvuu]+\frac{1}{4}s^2\,CW[tvuu]-\frac{7\,CW[tvuv]}{4}+\frac{11}{3}s\,CW[tvuv]+$$
$$\frac{7\,CW[tvvu]}{8}-\frac{11}{6}s\,CW[tvvu]-\frac{11\,CW[tvvv]}{12}-\frac{37\,CW[uuuv]}{24}+\frac{11}{12}s\,CW[uuuv]-$$
$$\frac{1}{4}s^2\,CW[uuuv]-\frac{4\,CW[uuvv]}{3}-\frac{2}{3}s\,CW[uuvv]+\frac{29\,CW[uvuv]}{12}+\frac{4}{3}s\,CW[uvuv]+\frac{25\,CW[uvvv]}{24} ==$$
$$\frac{25\,CW[tttu]}{24}-\frac{13\,CW[tttv]}{6}+\frac{11\,CW[ttuu]}{12}+\frac{5\,CW[ttuv]}{24}-\frac{11\,CW[ttvu]}{24}-\frac{19}{12}s\,CW[ttvu]+$$
$$\frac{29\,CW[ttvv]}{24}-\frac{25\,CW[tutu]}{12}+\frac{5\,CW[tutv]}{12}+\frac{3}{4}s\,CW[tutv]+\frac{1}{4}s^2\,CW[tutv]-\frac{11\,CW[tuuu]}{12}+$$
$$\frac{2\,CW[tuuv]}{3}-\frac{3}{4}s\,CW[tuuv]-\frac{1}{4}s^2\,CW[tuuv]-\frac{17\,CW[tuvu]}{4}+\frac{5}{4}s\,CW[tuvu]+$$
$$\frac{7\,CW[tuvv]}{4}-\frac{8\,CW[tvtv]}{3}+\frac{11\,CW[tvuu]}{8}-\frac{5}{12}s\,CW[tvuu]+\frac{1}{4}s^2\,CW[tvuu]-$$
$$\frac{7\,CW[tvuv]}{4}+\frac{11}{3}s\,CW[tvuv]+\frac{7\,CW[tvvu]}{8}-\frac{25}{12}s\,CW[tvvu]+\frac{1}{4}s^2\,CW[tvvu]-$$
$$\frac{11\,CW[tvvv]}{12}-\frac{37\,CW[uuuv]}{24}+\frac{11}{12}s\,CW[uuuv]-\frac{1}{4}s^2\,CW[uuuv]-\frac{4\,CW[uuvv]}{3}-$$
$$\frac{5}{12}s\,CW[uuvv]-\frac{1}{4}s^2\,CW[uuvv]+\frac{29\,CW[uvuv]}{12}+\frac{4}{3}s\,CW[uvuv]+\frac{25\,CW[uvvv]}{24}$$

```
Print /@ {
   0 → {α, β},
   1 → (t1 = Ju[α]),
   2 → (t2 = Ju[α // RCv[β]] // CCv[-β]),
   3 → t1 ≡ t2
  };
```

$$0 \to \left\{\mathrm{LS}\left[-t+2v,\ \overline{tu}-\frac{3\,\overline{tv}}{2}-\frac{\overline{uv}}{2},\right.\right.$$
$$\left.\frac{5}{6}\,\overline{t\,\overline{tu}}-\frac{5}{3}\,\overline{t\,\overline{tv}}+\frac{1}{2}\,\overline{t\,\overline{uv}}-\frac{7}{6}\,\overline{u\,\overline{uv}}+\frac{2}{3}\,\overline{\overline{tu}\,u}+\frac{5}{6}\,\overline{\overline{tv}\,u}-\frac{3}{2}\,\overline{\overline{tv}\,v}+\frac{1}{6}\,\overline{\overline{uv}\,v}\right],\ \mathrm{LS}\left[-t-u-2v,\right.$$
$$\left.\left.-\frac{\overline{tu}}{2}+\frac{3\,\overline{tv}}{2}+\overline{uv},\ \frac{4}{3}\,\overline{t\,\overline{tu}}-\overline{t\,\overline{tv}}+\frac{7}{6}\,\overline{t\,\overline{uv}}-\frac{2}{3}\,\overline{u\,\overline{uv}}+\frac{5}{3}\,\overline{\overline{tu}\,u}+\frac{1}{6}\,\overline{\overline{tv}\,u}+\frac{5}{3}\,\overline{\overline{tv}\,v}-\overline{\overline{uv}\,v}\right]\right\}$$

$$1 \to \mathrm{CWS}\left[0,\ \mathrm{CW}[tu]+\frac{\mathrm{CW}[uv]}{2},\ \frac{\mathrm{CW}[ttu]}{3}-\frac{2\,\mathrm{CW}[tuu]}{3}+\frac{\mathrm{CW}[tuv]}{6}+\frac{\mathrm{CW}[tvu]}{12}+\frac{7\,\mathrm{CW}[uuv]}{6}+\frac{2\,\mathrm{CW}[uvv]}{3}\right]$$

$$2 \to \mathrm{CWS}\left[0,\ \mathrm{CW}[tu]+\frac{5\,\mathrm{CW}[uv]}{2},\ \frac{\mathrm{CW}[ttu]}{3}-\frac{2\,\mathrm{CW}[tuu]}{3}+\frac{\mathrm{CW}[tuv]}{6}-\frac{29\,\mathrm{CW}[tvu]}{12}-\frac{\mathrm{CW}[uuv]}{3}+\frac{20\,\mathrm{CW}[uvv]}{3}\right]$$

$$3 \to \mathrm{CW}[tu]+\frac{\mathrm{CW}[uv]}{2} == \mathrm{CW}[tu]+\frac{5\,\mathrm{CW}[uv]}{2}$$

```
Print /@ {
    0 → {α, β},
    1 → (t1 = Ju[α] // RCu[α] // RCv[β] // RCu[α]),
    2 → (t2 = Ju[α // RCv[β]] // RCu[α // RCv[β]]),
    3 → t1 ≡ t2
  };
```

$$0 \to \left\{\mathrm{LS}\left[-t+2v,\ \overline{tu}-\frac{3\,\overline{tv}}{2}-\frac{\overline{uv}}{2},\right.\right.$$
$$\left.\frac{5}{6}\,\overline{t\,\overline{tu}}-\frac{5}{3}\,\overline{t\,\overline{tv}}+\frac{1}{2}\,\overline{t\,\overline{uv}}-\frac{7}{6}\,\overline{u\,\overline{uv}}+\frac{2}{3}\,\overline{\overline{tu}\,u}+\frac{5}{6}\,\overline{\overline{tv}\,u}-\frac{3}{2}\,\overline{\overline{tv}\,v}+\frac{1}{6}\,\overline{\overline{uv}\,v}\right],\ \mathrm{LS}\left[-t-u-2v,\right.$$
$$\left.\left.-\frac{\overline{tu}}{2}+\frac{3\,\overline{tv}}{2}+\overline{uv},\ \frac{4}{3}\,\overline{t\,\overline{tu}}-\overline{t\,\overline{tv}}+\frac{7}{6}\,\overline{t\,\overline{uv}}-\frac{2}{3}\,\overline{u\,\overline{uv}}+\frac{5}{3}\,\overline{\overline{tu}\,u}+\frac{1}{6}\,\overline{\overline{tv}\,u}+\frac{5}{3}\,\overline{\overline{tv}\,v}-\overline{\overline{uv}\,v}\right]\right\}$$

$$1 \to \mathrm{CWS}\left[0,\ \mathrm{CW}[tu]+\frac{\mathrm{CW}[uv]}{2},\ \frac{\mathrm{CW}[ttu]}{3}-\frac{2\,\mathrm{CW}[tuu]}{3}-\frac{13\,\mathrm{CW}[tuv]}{3}+\frac{55\,\mathrm{CW}[tvu]}{12}+\frac{7\,\mathrm{CW}[uuv]}{6}+\frac{2\,\mathrm{CW}[uvv]}{3}\right]$$

$$2 \to \mathrm{CWS}\left[0,\ \mathrm{CW}[tu]+\frac{5\,\mathrm{CW}[uv]}{2},\ \frac{\mathrm{CW}[ttu]}{3}-\frac{2\,\mathrm{CW}[tuu]}{3}-\frac{11\,\mathrm{CW}[tuv]}{6}-\frac{5\,\mathrm{CW}[tvu]}{12}-\frac{\mathrm{CW}[uuv]}{3}+\frac{20\,\mathrm{CW}[uvv]}{3}\right]$$

$$3 \to \mathrm{CW}[tu]+\frac{\mathrm{CW}[uv]}{2} == \mathrm{CW}[tu]+\frac{5\,\mathrm{CW}[uv]}{2}$$

3. The t equation

```
Print /@ {
    0 → γ,
    1 → (t1 = J[w, γ // tm[u, v, w]]),
    2 → (t2 = Ju[γ] // tm[u, v, w]),
    3 → (t3 = Jv[γ // RCu[γ]] // CCu[-γ] // tm[u, v, w]),
    4 → t1 ≡ t2 + t3
  };
```

$$0 \to \text{LS}\left[t - u - 2v,\ -\overline{tu} - 2\overline{tv} - 2\overline{uv},\ -\frac{5}{6}\overline{t\overline{tu}} - \overline{t\overline{tv}} + \frac{7}{6}\overline{t\overline{uv}} + \frac{1}{6}\overline{u\overline{uv}} - \frac{1}{2}\overline{\overline{tu}u} + 2\overline{\overline{tv}u} + \frac{2}{3}\overline{\overline{tv}v} + \frac{7}{6}\overline{\overline{uv}v}\right]$$

$$1 \to \text{CWS}\left[-3\,\text{CW}[w],\ -\frac{9\,\text{CW}[tw]}{2},\ -\frac{23\,\text{CW}[ttw]}{6} - \frac{49\,\text{CW}[tww]}{6}\right]$$

$$2 \to \text{CWS}\left[-\text{CW}[w],\ -\frac{3\,\text{CW}[tw]}{2} + 3\,\text{CW}[ww],\ -\frac{3\,\text{CW}[ttw]}{2} + \frac{4\,\text{CW}[tww]}{3} - \frac{\text{CW}[www]}{3}\right]$$

$$3 \to \text{CWS}\left[-2\,\text{CW}[w],\ -3\,\text{CW}[tw] - 3\,\text{CW}[ww],\ -\frac{7\,\text{CW}[ttw]}{3} - \frac{19\,\text{CW}[tww]}{2} + \frac{\text{CW}[www]}{3}\right]$$

$$4 \to \text{True}$$

```
Print /@ {
   0 → {γ, γw = γ // tm[u, v, w]},
   1 → (t1 = J[w, γw] // RC_w[γw]),
   2 → (t2 = J_u[γ] // tm[u, v, w] // RC_w[γw]),
   3 → (t3 = J_v[γ // RC_u[γ]] // RC_v[γ // RC_u[γ]] // tm[u, v, w]),
   4 → t1 ≡ t2 + t3
  };
```

$$0 \to \left\{\text{LS}\left[t - 2v,\ -\frac{3\overline{tu}}{2} + \frac{3\overline{tv}}{2} - 2\overline{uv},\ -\frac{3}{2}\overline{t\overline{tu}} - \overline{t\overline{tv}} - \frac{1}{6}\overline{t\overline{uv}} - \frac{7}{6}\overline{u\overline{uv}} - \frac{1}{3}\overline{\overline{tu}u} + \frac{1}{3}\overline{\overline{tv}u} - \overline{\overline{tv}v} - \frac{7}{6}\overline{\overline{uv}v}\right],\ \text{LS}\left[t - 2w,\ 0,\ -\frac{5}{2}\overline{t\overline{tw}} - \overline{\overline{tw}w}\right]\right\}$$

$$1 \to \text{CWS}\left[-2\,\text{CW}[w],\ -\text{CW}[tw],\ -\frac{17\,\text{CW}[ttw]}{6} + \frac{\text{CW}[tww]}{3}\right]$$

$$2 \to \text{CWS}\left[0,\ -\frac{3\,\text{CW}[tw]}{2} + 2\,\text{CW}[ww],\ -\frac{9\,\text{CW}[ttw]}{4} + 3\,\text{CW}[tww] - 2\,\text{CW}[www]\right]$$

$$3 \to \text{CWS}\left[-2\,\text{CW}[w],\ \frac{\text{CW}[tw]}{2} - 2\,\text{CW}[ww],\ -\frac{7\,\text{CW}[ttw]}{12} - \frac{8\,\text{CW}[tww]}{3} + 2\,\text{CW}[www]\right]$$

$$4 \to \text{True}$$

- Splitting the t equation

```
Print /@ {
   0 → {γ, γw = γ // tm[u, v, w]},
   1 → (t1 = γw // RC_w[s γw]),
   2 → (t2 = γ // RC_u[s γ] // RC_v[s γ // RC_u[s γ]] // tm[u, v, w]),
   3 → t1 ≡ t2,
   4 → (t3 = γ // RC_u[s γ] // RC_v[s γ // RC_u[s γ]] // tm[u, v, w] // div_w),
   5 → (t4 = γ // RC_u[s γ] // RC_v[s γ // RC_u[s γ]] // div_u // tm[u, v, w]),
   6 → (t5 = γ // RC_u[s γ] // RC_v[s γ // RC_u[s γ]] // div_v // tm[u, v, w]),
   7 → t3 ≡ t4 + t5,
   8 → (t6 = ∫_0^1 (t3 // CC_w[-s γw]) ⅆs),
   9 → t6 ≡ J_w[γw],
   10 → (t7 = ∫_0^1 (t4 // CC_w[-s γw]) ⅆs),
   11 → (t8 = ∫_0^1 (t5 // CC_w[-s γw]) ⅆs),
   12 → t6 ≡ t7 + t8,
   (J_u[γ] // tm[u, v, w] // RC_w[γw]) ≡ (t7 // RC_w[γw])
  };
```

$$0 \to \left\{ LS\left[-2\,t + 2\,u + v,\ \frac{\overline{t\,u}}{2} + \overline{t\,v} + \overline{u\,v},\right.\right.$$
$$\left. -\frac{7}{6}\,\overline{t\,\overline{t\,u}} - \frac{1}{6}\,\overline{t\,\overline{t\,v}} + \frac{7}{6}\,\overline{t\,\overline{u\,v}} - \frac{7}{6}\,\overline{u\,\overline{u\,v}} - 2\,\overline{\overline{t\,u}\,u} - \frac{3}{2}\,\overline{\overline{t\,v}\,u} + \frac{2}{3}\,\overline{\overline{t\,v}\,v} + \overline{\overline{u\,v}\,v} \right],$$
$$\left. LS\left[-2\,t + 3\,w,\ \frac{3\,\overline{t\,w}}{2},\ -\frac{4}{3}\,\overline{t\,\overline{t\,w}} - \frac{17}{6}\,\overline{\overline{t\,w}\,w} \right] \right\}$$

$$1 \to LS\left[-2\,t + 3\,w,\ \frac{3\,\overline{t\,w}}{2} - 6\,s\,\overline{t\,w},\ -\frac{4}{3}\,\overline{t\,\overline{t\,w}} - 3\,s\,\overline{t\,\overline{t\,w}} + 6\,s^2\,\overline{t\,\overline{t\,w}} - \frac{17}{6}\,\overline{\overline{t\,w}\,w} + \frac{9}{2}\,s\,\overline{\overline{t\,w}\,w} - 9\,s^2\,\overline{\overline{t\,w}\,w} \right]$$

$$2 \to LS\left[-2\,t + 3\,w,\ \frac{3\,\overline{t\,w}}{2} - 6\,s\,\overline{t\,w},\ -\frac{4}{3}\,\overline{t\,\overline{t\,w}} - 3\,s\,\overline{t\,\overline{t\,w}} + 6\,s^2\,\overline{t\,\overline{t\,w}} - \frac{17}{6}\,\overline{\overline{t\,w}\,w} + \frac{9}{2}\,s\,\overline{\overline{t\,w}\,w} - 9\,s^2\,\overline{\overline{t\,w}\,w} \right]$$

$$3 \to \text{True}$$

$$4 \to CWS\left[3\,CW[w],\ \frac{3\,CW[tw]}{2} - 6\,s\,CW[tw],\right.$$
$$\left. -\frac{4\,CW[ttw]}{3} - 3\,s\,CW[ttw] + 6\,s^2\,CW[ttw] + \frac{17\,CW[tww]}{6} - \frac{9}{2}\,s\,CW[tww] + 9\,s^2\,CW[tww] \right]$$

$$5 \to CWS\left[2\,CW[w],\ \frac{CW[tw]}{2} - 4\,s\,CW[tw] - CW[ww],\right.$$
$$\left. -\frac{7\,CW[ttw]}{6} - s\,CW[ttw] + 4\,s^2\,CW[ttw] + \frac{5\,CW[tww]}{6} - s\,CW[tww] + 6\,s^2\,CW[tww] + \frac{13\,CW[www]}{6} \right]$$

$$6 \to CWS\left[CW[w],\ CW[tw] - 2\,s\,CW[tw] + CW[ww],\right.$$
$$\left. -\frac{CW[ttw]}{6} - 2\,s\,CW[ttw] + 2\,s^2\,CW[ttw] + 2\,CW[tww] - \frac{7}{2}\,s\,CW[tww] + 3\,s^2\,CW[tww] - \frac{13\,CW[www]}{6} \right]$$

$$7 \to \text{True}$$

$$8 \to CWS\left[3\,CW[w],\ -\frac{3\,CW[tw]}{2},\ -\frac{5\,CW[ttw]}{6} + \frac{43\,CW[tww]}{12} \right]$$

$$9 \to \text{True}$$

$$10 \to CWS\left[2\,CW[w],\ -\frac{3\,CW[tw]}{2} - CW[ww],\ -\frac{CW[ttw]}{3} + \frac{7\,CW[tww]}{3} + \frac{13\,CW[www]}{6} \right]$$

$$11 \to CWS\left[CW[w],\ CW[ww],\ -\frac{CW[ttw]}{2} + \frac{5\,CW[tww]}{4} - \frac{13\,CW[www]}{6} \right]$$

$$12 \to \text{True}$$

$$-\frac{3\,CW[tw]}{2} == -\frac{3\,CW[tw]}{2} - CW[ww]$$

4. The h equation

```
Print /@ {
   1 → (t1 = J[u, BCH[α, β]]),
   2 → (t2 = J[u, α]),
   3 → (t3 = J[u, β // RC[u, α]] // CC[u, -α]),
   4 → t1 ≡ t2 + t3
  };
```

$$1 \to \text{CWS}\left[3\,\text{CW}[u],\ -\frac{7\,\text{CW}[tu]}{2}+\frac{5\,\text{CW}[uv]}{2},\ \frac{11\,\text{CW}[ttu]}{6}+\frac{\text{CW}[tuu]}{2}-\frac{25\,\text{CW}[tuv]}{6}+\frac{17\,\text{CW}[tvu]}{12}-\frac{25\,\text{CW}[uuv]}{6}-\frac{9\,\text{CW}[uvv]}{4}\right]$$

$$2 \to \text{CWS}\left[2\,\text{CW}[u],\ -\text{CW}[tu]+\frac{3\,\text{CW}[uv]}{2},\ \frac{\text{CW}[ttu]}{3}-\frac{\text{CW}[tuu]}{3}-\frac{7\,\text{CW}[tuv]}{6}+\frac{17\,\text{CW}[tvu]}{12}-\frac{\text{CW}[uuv]}{3}-\frac{5\,\text{CW}[uvv]}{4}\right]$$

$$3 \to \text{CWS}\left[\text{CW}[u],\ -\frac{5\,\text{CW}[tu]}{2}+\text{CW}[uv],\ \frac{3\,\text{CW}[ttu]}{2}+\frac{5\,\text{CW}[tuu]}{6}-3\,\text{CW}[tuv]-\frac{23\,\text{CW}[uuv]}{6}-\text{CW}[uvv]\right]$$

$$4 \to \text{True}$$

- h and S

```
(Plus[
    Ju[γ] // RCu[γ],
    Ju[-γ // RCu[γ]]
   ] // RCu[-γ // RCu[γ]]) @ {6}
```

$$\text{CWS}[0,\ 0,\ 0,\ 0,\ 0,\ 0]$$

5. The meaning(s) of RC

```
Print /@ {
   1 → α,
   2 → (t1 = α // CC[u, γ] // RC[u, -γ]),
   3 → α ≡ t1
  };
```

$$1 \to \text{LS}\left[2\,t-v,\ -\frac{\overline{tu}}{2}+\overline{tv}-2\,\overline{uv},\ -\frac{5}{3}\,\overline{t\,\overline{tu}}+\frac{4}{3}\,\overline{t\,\overline{tv}}-\frac{3}{2}\,\overline{t\,\overline{uv}}+\frac{1}{3}\,\overline{u\,\overline{uv}}-\frac{1}{6}\,\overline{\overline{tu}\,u}-\overline{\overline{tv}\,u}+\overline{\overline{tv}\,v}+2\,\overline{\overline{uv}\,v}\right]$$

$$2 \to \text{LS}\left[2\,t-v,\ -\frac{\overline{tu}}{2}+\overline{tv}-2\,\overline{uv},\ -\frac{5}{3}\,\overline{t\,\overline{tu}}+\frac{4}{3}\,\overline{t\,\overline{tv}}-\frac{3}{2}\,\overline{t\,\overline{uv}}+\frac{1}{3}\,\overline{u\,\overline{uv}}-\frac{1}{6}\,\overline{\overline{tu}\,u}-\overline{\overline{tv}\,u}+\overline{\overline{tv}\,v}+2\,\overline{\overline{uv}\,v}\right]$$

$$3 \to \text{True}$$

```
Print /@ {
   1 → α,
   2 → (t1 = α // CC[u, γ // RC[u, γ]]),
   3 → (t2 = α // RC[u, γ]),
   4 → t1 ≡ t2
  };
```

$$1 \to LS\left[2t-v,\ -\frac{\overline{tu}}{2}+\overline{tv}-2\overline{uv},\ -\frac{5}{3}\overline{t\overline{tu}}+\frac{4}{3}\overline{t\overline{tv}}-\frac{3}{2}\overline{t\overline{uv}}+\frac{1}{3}\overline{u\overline{uv}}-\frac{1}{6}\overline{\overline{tu}u}-\overline{\overline{tv}u}+\overline{\overline{tv}v}+2\overline{\overline{uv}v}\right]$$

$$2 \to LS\left[2t-v,\ -\frac{\overline{tu}}{2}+\overline{tv}-2\overline{uv},\ -\frac{13}{6}\overline{t\overline{tu}}+\frac{4}{3}\overline{t\overline{tv}}-3\overline{t\overline{uv}}+\frac{1}{3}\overline{u\overline{uv}}-\frac{1}{6}\overline{\overline{tu}u}-3\overline{\overline{tv}u}+\overline{\overline{tv}v}+4\overline{\overline{uv}v}\right]$$

$$3 \to LS\left[2t-v,\ -\frac{\overline{tu}}{2}+\overline{tv}-2\overline{uv},\ -\frac{13}{6}\overline{t\overline{tu}}+\frac{4}{3}\overline{t\overline{tv}}-3\overline{t\overline{uv}}+\frac{1}{3}\overline{u\overline{uv}}-\frac{1}{6}\overline{\overline{tu}u}-3\overline{\overline{tv}u}+\overline{\overline{tv}v}+4\overline{\overline{uv}v}\right]$$

$$4 \to \text{True}$$

6. $C_u\,C_v$ and $\mathrm{RC}_u\,\mathrm{RC}_v$

```
Print /@ {
    1 → {α, β, γ},
    2 → (t1 = γ // CCu[α // RCv[-β]] // CCv[β]),
    3 → (t2 = γ // CCv[β // RCu[-α]] // CCu[α]),
    4 → t1 ≡ t2
  };
```

$$1 \to \left\{LS\left[-v,\ -\frac{3\overline{tu}}{2}-\overline{tv}+\overline{uv},\ \overline{t\overline{tu}}+\frac{3}{2}\overline{t\overline{tv}}+\frac{1}{2}\overline{t\overline{uv}}+\frac{2}{3}\overline{u\overline{uv}}-2\overline{\overline{tu}u}-\frac{5}{6}\overline{\overline{tv}u}-\frac{5}{6}\overline{\overline{tv}v}+2\overline{\overline{uv}v}\right],\right.$$
$$LS\left[2u+v,\ -2\overline{tv}+2\overline{uv},\ \frac{11}{6}\overline{t\overline{tu}}+\frac{3}{2}\overline{t\overline{tv}}+\frac{5}{6}\overline{t\overline{uv}}+2\overline{u\overline{uv}}-\frac{2}{3}\overline{\overline{tu}u}+\frac{4}{3}\overline{\overline{tv}u}-\frac{7}{6}\overline{\overline{tv}v}-2\overline{\overline{uv}v}\right],$$
$$\left.LS\left[-t-2u+v,\ \frac{3\overline{tu}}{2}-\overline{tv}-\overline{uv},\ -\frac{1}{6}\overline{t\overline{tv}}-\frac{3}{2}\overline{t\overline{uv}}-\frac{4}{3}\overline{u\overline{uv}}+\frac{5}{3}\overline{\overline{tu}u}-\frac{2}{3}\overline{\overline{tv}u}+\frac{1}{6}\overline{\overline{tv}v}+\frac{1}{6}\overline{\overline{uv}v}\right]\right\}$$

$$2 \to LS\left[-t-2u+v,\ \frac{3\overline{tu}}{2}-\overline{tv}-\overline{uv},\ -\frac{1}{6}\overline{t\overline{tv}}-2\overline{t\overline{uv}}+\frac{2}{3}\overline{u\overline{uv}}+\frac{14}{3}\overline{\overline{tu}u}+\frac{4}{3}\overline{\overline{tv}u}-\frac{11}{6}\overline{\overline{tv}v}-\frac{5}{6}\overline{\overline{uv}v}\right]$$

$$3 \to LS\left[-t-2u+v,\ \frac{3\overline{tu}}{2}-\overline{tv}-\overline{uv},\ -\frac{1}{6}\overline{t\overline{tv}}-2\overline{t\overline{uv}}+\frac{2}{3}\overline{u\overline{uv}}+\frac{14}{3}\overline{\overline{tu}u}+\frac{4}{3}\overline{\overline{tv}u}-\frac{11}{6}\overline{\overline{tv}v}-\frac{5}{6}\overline{\overline{uv}v}\right]$$

$$4 \to \text{True}$$

```
Print /@ {
    1 → {α, β, γ},
    2 → (t1 = γ // RCu[α] // RCv[β // RCu[α]]),
    3 → (t2 = γ // RCv[β] // RCu[α // RCv[β]]),
    4 → t1 ≡ t2
  };
```

$$1 \to \left\{ LS\left[-v,\ -\frac{3\,\overline{tu}}{2} - \overline{tv} + \overline{uv},\ \overline{t\,\overline{tu}} + \frac{3}{2}\,\overline{t\,\overline{tv}} + \frac{1}{2}\,\overline{t\,\overline{uv}} + \frac{2}{3}\,\overline{u\,\overline{uv}} - 2\,\overline{tu\,u} - \frac{5}{6}\,\overline{tv\,u} - \frac{5}{6}\,\overline{tv\,v} + 2\,\overline{uv\,v} \right], \right.$$

$$LS\left[2u + v,\ -2\,\overline{tv} + 2\,\overline{uv},\right.$$

$$\left. \frac{11}{6}\,\overline{t\,\overline{tu}} + \frac{3}{2}\,\overline{t\,\overline{tv}} + \frac{5}{6}\,\overline{t\,\overline{uv}} + 2\,\overline{u\,\overline{uv}} - \frac{2}{3}\,\overline{tu\,u} + \frac{4}{3}\,\overline{tv\,u} - \frac{7}{6}\,\overline{tv\,v} - 2\,\overline{uv\,v} \right],$$

$$\left. LS\left[-t - 2u + v,\ \frac{3\,\overline{tu}}{2} - \overline{tv} - \overline{uv},\ -\frac{1}{6}\,\overline{t\,\overline{tv}} - \frac{3}{2}\,\overline{t\,\overline{uv}} - \frac{4}{3}\,\overline{u\,\overline{uv}} + \frac{5}{3}\,\overline{tu\,u} - \frac{2}{3}\,\overline{tv\,u} + \frac{1}{6}\,\overline{tv\,v} + \frac{1}{6}\,\overline{uv\,v} \right] \right\}$$

$$2 \to LS\left[-t - 2u + v,\ \frac{3\,\overline{tu}}{2} - \overline{tv} - \overline{uv},\right.$$

$$\left. -\frac{1}{6}\,\overline{t\,\overline{tv}} - 2\,\overline{t\,\overline{uv}} - \frac{10}{3}\,\overline{u\,\overline{uv}} + \frac{14}{3}\,\overline{tu\,u} + \frac{4}{3}\,\overline{tv\,u} - \frac{11}{6}\,\overline{tv\,v} + \frac{19}{6}\,\overline{uv\,v} \right]$$

$$3 \to LS\left[-t - 2u + v,\ \frac{3\,\overline{tu}}{2} - \overline{tv} - \overline{uv},\right.$$

$$\left. -\frac{1}{6}\,\overline{t\,\overline{tv}} - 2\,\overline{t\,\overline{uv}} - \frac{10}{3}\,\overline{u\,\overline{uv}} + \frac{14}{3}\,\overline{tu\,u} + \frac{4}{3}\,\overline{tv\,u} - \frac{11}{6}\,\overline{tv\,v} + \frac{19}{6}\,\overline{uv\,v} \right]$$

$$4 \to \text{True}$$

7.

8.

9.

10.

11. div property uv

```
Print /@ {
   0 → {α, β},
   1 → (t1 = divu[α] // adv[β]),
   2 → (t2 = divv[β] // adu[α]),
   3 → (t3 = MakeCWSeries[0]),
   4 → (t4 = divu[α // adv[β]]),
   5 → (t5 = divv[β // adu[α]]),
   6 → t1 - t2 ≡ t3 + t4 - t5
  };
```

$$0 \to \left\{\mathrm{LS}\left[t+2u+v,\ -\frac{3\,\overline{tu}}{2}+\overline{tv}+\frac{\overline{uv}}{2},\right.\right.$$
$$\left.-\frac{5}{6}\,\overline{t\,\overline{tu}}-\frac{11}{6}\,\overline{t\,\overline{tv}}-\frac{3}{2}\,\overline{t\,\overline{uv}}-\frac{1}{6}\,\overline{u\,\overline{uv}}+\frac{1}{2}\,\overline{\overline{tu}\,u}+\frac{3}{2}\,\overline{\overline{tv}\,u}+\frac{11}{6}\,\overline{\overline{tv}\,v}+2\,\overline{\overline{uv}\,v}\right],\ \mathrm{LS}\left[-t+u-v,\right.$$
$$\left.\left.2\,\overline{tv}-\frac{3\,\overline{uv}}{2},\ -\frac{5}{3}\,\overline{t\,\overline{tu}}+\frac{4}{3}\,\overline{t\,\overline{tv}}+\frac{7}{6}\,\overline{t\,\overline{uv}}+\frac{1}{6}\,\overline{u\,\overline{uv}}-\frac{1}{2}\,\overline{\overline{tu}\,u}+\frac{2}{3}\,\overline{\overline{tv}\,u}+\frac{2}{3}\,\overline{\overline{tv}\,v}+\overline{\overline{uv}\,v}\right]\right\}$$

$$1 \to \mathrm{CWS}\left[0,\ 0,\ -\frac{\mathrm{CW}[tuv]}{2}+\frac{\mathrm{CW}[tvu]}{2}\right]$$

$$2 \to \mathrm{CWS}\left[0,\ 0,\ -\frac{3\,\mathrm{CW}[tuv]}{2}+\frac{3\,\mathrm{CW}[tvu]}{2}\right]$$

$$3 \to \mathrm{CWS}[0,\ 0,\ 0]$$

$$4 \to \mathrm{CWS}\left[0,\ -\mathrm{CW}[uv],\ -\frac{\mathrm{CW}[tuv]}{2}-\frac{\mathrm{CW}[tvu]}{2}-\frac{\mathrm{CW}[uuv]}{2}-\frac{3\,\mathrm{CW}[uvv]}{2}\right]$$

$$5 \to \mathrm{CWS}\left[0,\ -\mathrm{CW}[uv],\ -\frac{3\,\mathrm{CW}[tuv]}{2}+\frac{\mathrm{CW}[tvu]}{2}-\frac{\mathrm{CW}[uuv]}{2}-\frac{3\,\mathrm{CW}[uvv]}{2}\right]$$

$$6 \to \mathrm{True}$$

12. div property uu

```
Print /@ {
    0 → {α, β},
    1 → (t1 = divu[α] // adu[β]),
    2 → (t2 = divu[β] // adu[α]),
    3 → (t3 = divu[b[α, β]]),
    4 → (t4 = divu[α // adu[β]]),
    5 → (t5 = divu[β // adu[α]]),
    6 → t1 - t2 ≡ t3 + t4 - t5
  };
```

$$0 \to \left\{\mathrm{LS}\left[-2u+2v,\ -\frac{\overline{tu}}{2}-\frac{\overline{tv}}{2}-\frac{3\,\overline{uv}}{2},\right.\right.$$
$$\left.-\frac{1}{3}\,\overline{t\,\overline{tu}}+\frac{1}{3}\,\overline{t\,\overline{tv}}-\frac{1}{6}\,\overline{t\,\overline{uv}}+\frac{7}{6}\,\overline{u\,\overline{uv}}+\frac{5}{3}\,\overline{\overline{tu}\,u}-\frac{1}{3}\,\overline{\overline{tv}\,u}-\frac{7}{6}\,\overline{\overline{tv}\,v}-\overline{\overline{uv}\,v}\right],\ \mathrm{LS}\left[t-u-2v,\right.$$
$$\left.\left.\frac{3\,\overline{tu}}{2}-\frac{3\,\overline{tv}}{2}+\frac{3\,\overline{uv}}{2},\ \frac{1}{6}\,\overline{t\,\overline{tu}}+\frac{1}{6}\,\overline{t\,\overline{tv}}+\frac{4}{3}\,\overline{t\,\overline{uv}}-\frac{5}{3}\,\overline{u\,\overline{uv}}+\frac{3}{2}\,\overline{\overline{tu}\,u}-\frac{1}{2}\,\overline{\overline{tv}\,u}-\frac{1}{6}\,\overline{\overline{tv}\,v}-\frac{4}{3}\,\overline{\overline{uv}\,v}\right]\right\}$$

$$1 \to \mathrm{CWS}\left[0,\ 0,\ \frac{\mathrm{CW}[tuv]}{2}-\frac{\mathrm{CW}[tvu]}{2}\right]$$

$$2 \to \mathrm{CWS}[0,\ 0,\ -3\,\mathrm{CW}[tuv]+3\,\mathrm{CW}[tvu]]$$

$$3 \to \mathrm{CWS}\left[0,\ 2\,\mathrm{CW}[tu]-6\,\mathrm{CW}[uv],\ \frac{\mathrm{CW}[ttu]}{2}-\frac{7\,\mathrm{CW}[tuu]}{2}+\frac{9\,\mathrm{CW}[tuv]}{2}-4\,\mathrm{CW}[tvu]+\frac{9\,\mathrm{CW}[uuv]}{2}\right]$$

$$4 \to \mathrm{CWS}\left[0,\ -2\,\mathrm{CW}[tu]+4\,\mathrm{CW}[uv],\ -\frac{\mathrm{CW}[ttu]}{2}+3\,\mathrm{CW}[tuu]-\frac{3\,\mathrm{CW}[tuv]}{2}+4\,\mathrm{CW}[tvu]-3\,\mathrm{CW}[uuv]-3\,\mathrm{CW}[uvv]\right]$$

$$5 \to \mathrm{CWS}\left[0,\ -2\,\mathrm{CW}[uv],\ -\frac{\mathrm{CW}[tuu]}{2}-\frac{\mathrm{CW}[tuv]}{2}+\frac{7\,\mathrm{CW}[tvu]}{2}+\frac{3\,\mathrm{CW}[uuv]}{2}-3\,\mathrm{CW}[uvv]\right]$$

$$6 \to \mathrm{True}$$

13.

14.

15.

16.

17.

18.

19. The differential of BCH

```
Print /@ {
    1 → (bch = BCH[u, v]),
    2 → (BCH[u + ϵ t, v + ϵ w] - bch)/ϵ,
    3 → (t1 = (BCH[u + ϵ t, v + ϵ w] - bch)/ϵ // adSeries[(1 - e^-ad)/ad, bch]),
    4 → (t2 = t // adSeries[(1 - e^-ad)/ad, u] // Ad[-v]),
    5 → (t3 = w // adSeries[(1 - e^-ad)/ad, v])
  };
t1 ≡ t2 + t3
```

$$1 \to \mathrm{LS}\left[u+v,\ \frac{\overline{uv}}{2},\ \frac{1}{12}\overline{u\overline{uv}}+\frac{1}{12}\overline{\overline{uv}v}\right]$$

$$2 \to \mathrm{LS}\left[t+w,\ \frac{\overline{tv}}{2}+\frac{\overline{uw}}{2},\ \frac{1}{12}\overline{t\overline{uv}}+\frac{1}{12}\overline{u\overline{uw}}+\frac{1}{12}\overline{u\overline{vw}}-\frac{1}{12}\overline{\overline{tv}u}+\frac{1}{12}\overline{\overline{tv}v}+\frac{1}{6}\overline{\overline{uw}v}\right]$$

$$3 \to \mathrm{LS}\left[t+w,\ \frac{\overline{tu}}{2}+\overline{tv}-\frac{\overline{vw}}{2},\ \frac{1}{2}\overline{t\overline{uv}}+\frac{1}{6}\overline{v\overline{vw}}+\frac{1}{6}\overline{\overline{tu}u}+\frac{1}{2}\overline{\overline{tv}u}+\frac{1}{2}\overline{\overline{tv}v}\right]$$

$$4 \to \mathrm{LS}\left[t,\ \frac{\overline{tu}}{2}+\overline{tv},\ \frac{1}{2}\overline{t\overline{uv}}+\frac{1}{6}\overline{\overline{tu}u}+\frac{1}{2}\overline{\overline{tv}u}+\frac{1}{2}\overline{\overline{tv}v}\right]$$

$$5 \to \mathrm{LS}\left[w,\ -\frac{\overline{vw}}{2},\ \frac{1}{6}\overline{v\overline{vw}}\right]$$

True

20. The differential of C

```
Print /@ {
    0 → {α, β, γ},
    1 → (t1 = ((γ // CC[u, α + ϵ β]) - (γ // CC[u, α])) / ϵ),
    2 → (t2 = γ // ad[u, adSeries[(e^ad - 1)/ad, α][β] // RC[u, -α]] // CC[u, α]),
    t1 ≡ t2
  };
```

$$0 \to \Big\{ LS\Big[-2t-2u+2v,\ -\frac{\overline{tv}}{2}+\overline{uv},\ -\frac{4}{3}\overline{t\overline{tu}}-\frac{3}{2}\overline{t\overline{tv}}+\frac{3}{2}\overline{t\overline{uv}}+\frac{1}{6}\overline{u\overline{uv}}-\frac{5}{3}\overline{\overline{tu}u}-\frac{2}{3}\overline{\overline{tv}u}+\frac{5}{6}\overline{\overline{tv}v}+\frac{5}{3}\overline{\overline{uv}v}\Big],$$
$$LS\Big[2u+2v,\ 2\overline{tu}-\overline{tv}+\frac{3\overline{uv}}{2},\ \frac{1}{2}\overline{t\overline{tu}}+\frac{5}{3}\overline{t\overline{tv}}+\frac{1}{6}\overline{t\overline{uv}}+\frac{2}{3}\overline{u\overline{uv}}-\frac{7}{6}\overline{\overline{tu}u}-\frac{7}{6}\overline{\overline{tv}u}-2\overline{\overline{tv}v}+\frac{1}{6}\overline{\overline{uv}v}\Big],$$
$$LS\Big[t+u-v,\ 2\overline{tu}+\overline{tv}-\frac{3\overline{uv}}{2},\ -\frac{11}{6}\overline{t\overline{tu}}-\frac{1}{3}\overline{t\overline{tv}}-\frac{5}{6}\overline{t\overline{uv}}+2\overline{u\overline{uv}}+\frac{7}{6}\overline{\overline{tu}u}-\frac{5}{6}\overline{\overline{tv}u}-\frac{11}{6}\overline{\overline{tv}v}-\frac{11}{6}\overline{\overline{uv}v}\Big]\Big\}$$

$$1 \to LS\Big[0,\ -2\overline{uv},\ -\frac{3}{2}\overline{u\overline{uv}}+4\overline{\overline{tu}u}+\overline{\overline{tv}u}+7\overline{\overline{uv}v}\Big]$$

$$2 \to LS\Big[0,\ -2\overline{uv},\ -\frac{3}{2}\overline{u\overline{uv}}+4\overline{\overline{tu}u}+\overline{\overline{tv}u}+7\overline{\overline{uv}v}\Big]$$

True

21. The differential of RC

```
Print /@ {
    0 → {α, β, γ},
    1 → (t1 = ((γ // RC[u, α + ϵ β]) - (γ // RC[u, α])) / ϵ),
    2 → (t2 = γ // RC[u, α] // ad[u, adSeries[(1 - e^-ad)/ad, α][β] // RC[u, α]]),
    t1 ≡ t2
  };
```

$$0 \to \left\{ LS\left[-2t - 2u + 2v,\ -\frac{\overline{tv}}{2} + \overline{uv},\right.\right.$$
$$\left. -\frac{4}{3}\overline{t\overline{tu}} - \frac{3}{2}\overline{t\overline{tv}} + \frac{3}{2}\overline{t\overline{uv}} + \frac{1}{6}\overline{u\overline{uv}} - \frac{5}{3}\overline{\overline{tu}u} - \frac{2}{3}\overline{\overline{tv}u} + \frac{5}{6}\overline{\overline{tv}v} + \frac{5}{3}\overline{\overline{uv}v} \right],$$
$$LS\left[2u + 2v,\ 2\overline{tu} - \overline{tv} + \frac{3\overline{uv}}{2},\ \frac{1}{2}\overline{t\overline{tu}} + \frac{5}{3}\overline{t\overline{tv}} + \frac{1}{6}\overline{t\overline{uv}} + \frac{2}{3}\overline{u\overline{uv}} - \right.$$
$$\left. \frac{7}{6}\overline{\overline{tu}u} - \frac{7}{6}\overline{\overline{tv}u} - 2\overline{\overline{tv}v} + \frac{1}{6}\overline{\overline{uv}v} \right],\ LS\left[t + u - v,\ 2\overline{tu} + \overline{tv} - \frac{3\overline{uv}}{2},\right.$$
$$\left. \left. -\frac{11}{6}\overline{t\overline{tu}} - \frac{1}{3}\overline{t\overline{tv}} - \frac{5}{6}\overline{t\overline{uv}} + 2\overline{u\overline{uv}} + \frac{7}{6}\overline{\overline{tu}u} - \frac{5}{6}\overline{\overline{tv}u} - \frac{11}{6}\overline{\overline{tv}v} - \frac{11}{6}\overline{\overline{uv}v} \right] \right\}$$

$$1 \to LS\left[0,\ -2\overline{uv},\ -\frac{3}{2}\overline{u\overline{uv}} + \overline{\overline{tv}u} + 7\overline{\overline{uv}v} \right]$$

$$2 \to LS\left[0,\ -2\overline{uv},\ -\frac{3}{2}\overline{u\overline{uv}} + \overline{\overline{tv}u} + 7\overline{\overline{uv}v} \right]$$

True

22. The differential of J

```
Print /@ {
   0 → {α, β},
   1 → (t0 = (J[u, α + ϵ β] - J[u, α])/ϵ),
   2 → (t1 = ∫_0^1 (div[u, β // RC[u, s α]] // CC[u, -s α]) ⅆs),
   3 → (t2 = ∫_0^1 (s div[u, α // RC[u, s α] //
            ad[u, β // adSeries[(1 - ℯ^(-s ad))/(s ad), α] // RC[u, s α]]] // CC[u, -s α]) ⅆs),
   4 → (t3 = ∫_0^1 (s div[u, α // RC[u, s α]] // ad[u, β // adSeries[(1 - ℯ^(-s ad))/(s ad), α] //
            RC[u, s α]] // CC[u, -s α]) ⅆs),
   t0 ≡ t1 + t2 - t3
  };
```

$$0 \to \left\{\mathrm{LS}\left[-2\,t-2\,u+2\,v,\ -\frac{\overline{t\,v}}{2}+\overline{u\,v},\right.\right.$$
$$\left.-\frac{4}{3}\,\overline{t\,\overline{t\,u}}-\frac{3}{2}\,\overline{t\,\overline{t\,v}}+\frac{3}{2}\,\overline{t\,\overline{u\,v}}+\frac{1}{6}\,\overline{u\,\overline{u\,v}}-\frac{5}{3}\,\overline{\overline{t\,u}\,u}-\frac{2}{3}\,\overline{\overline{t\,v}\,u}+\frac{5}{6}\,\overline{\overline{t\,v}\,v}+\frac{5}{3}\,\overline{\overline{u\,v}\,v}\right],\ \mathrm{LS}\left[2\,u+2\,v,\right.$$
$$\left.\left.2\,\overline{t\,u}-\overline{t\,v}+\frac{3\,\overline{u\,v}}{2},\ \frac{1}{2}\,\overline{t\,\overline{t\,u}}+\frac{5}{3}\,\overline{t\,\overline{t\,v}}+\frac{1}{6}\,\overline{t\,\overline{u\,v}}+\frac{2}{3}\,\overline{u\,\overline{u\,v}}-\frac{7}{6}\,\overline{\overline{t\,u}\,u}-\frac{7}{6}\,\overline{\overline{t\,v}\,u}-2\,\overline{\overline{t\,v}\,v}+\frac{1}{6}\,\overline{\overline{u\,v}\,v}\right]\right\}$$

$$1 \to \mathrm{CWS}\left[2\,\mathrm{CW}[u],\ -\frac{3\,\mathrm{CW}[uv]}{2},\ -\frac{\mathrm{CW}[ttu]}{6}+\frac{\mathrm{CW}[tuu]}{2}+\frac{8\,\mathrm{CW}[tuv]}{3}+\frac{2\,\mathrm{CW}[tvu]}{3}+\frac{\mathrm{CW}[uuv]}{6}-\frac{11\,\mathrm{CW}[uvv]}{3}\right]$$

$$2 \to \mathrm{CWS}\left[2\,\mathrm{CW}[u],\ \frac{\mathrm{CW}[uv]}{2},\ -\frac{\mathrm{CW}[ttu]}{6}-\frac{\mathrm{CW}[tuu]}{6}+\frac{7\,\mathrm{CW}[tuv]}{3}-\frac{5\,\mathrm{CW}[tvu]}{3}+\frac{5\,\mathrm{CW}[uuv]}{3}\right]$$

$$3 \to \mathrm{CWS}\left[0,\ -2\,\mathrm{CW}[uv],\ \frac{2\,\mathrm{CW}[tuu]}{3}-\frac{7\,\mathrm{CW}[tuv]}{3}+5\,\mathrm{CW}[tvu]-\frac{3\,\mathrm{CW}[uuv]}{2}-\frac{11\,\mathrm{CW}[uvv]}{3}\right]$$

$$4 \to \mathrm{CWS}\left[0,\ 0,\ -\frac{8\,\mathrm{CW}[tuv]}{3}+\frac{8\,\mathrm{CW}[tvu]}{3}\right]$$

True

```
Print /@ {
   0 → {α, β};
   1 → (t0 = (J[u, α + ϵ β] - J[u, α])/ϵ),
   2 → (t1 = ∫_0^1 (div[u, β // RC[u, s α]] // CC[u, -s α]) ds),
   3 → (t2 = ∫_0^1 (div[u, β // adSeries[(1 - e^(-s ad))/ad, α] // RC[u, s α] //
           ad[u, α // RC[u, s α]]] // CC[u, -s α]) ds),
   4 → (t3 = ∫_0^1 (div[u, β // adSeries[(1 - e^(-s ad))/ad, α] // RC[u, s α]] //
         ad[u, α // RC[u, s α]] // CC[u, -s α]) ds),
   5 → (t4 = ∫_0^1 (div[u, b[α // RC[u, s α], β // adSeries[(1 - e^(-s ad))/ad, α] //
           RC[u, s α]]] // CC[u, -s α]) ds),
   t0 ≡ t1 + t2 - t3 - t4
  };
```

$$1 \to \text{CWS}\left[2\,\text{CW}[u],\ -\frac{3\,\text{CW}[uv]}{2},\ -\frac{\text{CW}[ttu]}{6}+\frac{\text{CW}[tuu]}{2}+\frac{8\,\text{CW}[tuv]}{3}+\frac{2\,\text{CW}[tvu]}{3}+\frac{\text{CW}[uuv]}{6}-\frac{11\,\text{CW}[uvv]}{3}\right]$$

$$2 \to \text{CWS}\left[2\,\text{CW}[u],\ \frac{\text{CW}[uv]}{2},\ -\frac{\text{CW}[ttu]}{6}-\frac{\text{CW}[tuu]}{6}+\frac{7\,\text{CW}[tuv]}{3}-\frac{5\,\text{CW}[tvu]}{3}+\frac{5\,\text{CW}[uuv]}{3}\right]$$

$$3 \to \text{CWS}\left[0,\ -2\,\text{CW}[tu]+2\,\text{CW}[uv],\ -\frac{2\,\text{CW}[ttu]}{3}-\frac{8\,\text{CW}[tuu]}{3}+2\,\text{CW}[tuv]+\frac{\text{CW}[tvu]}{6}+\frac{11\,\text{CW}[uuv]}{3}-\frac{3\,\text{CW}[uvv]}{2}\right]$$

$$4 \to \text{CWS}\left[0,\ 0,\ \frac{5\,\text{CW}[tuv]}{6}-\frac{5\,\text{CW}[tvu]}{6}\right]$$

$$5 \to \text{CWS}\left[0,\ -2\,\text{CW}[tu]+4\,\text{CW}[uv],\ -\frac{2\,\text{CW}[ttu]}{3}-\frac{10\,\text{CW}[tuu]}{3}+\frac{5\,\text{CW}[tuv]}{6}-\frac{4\,\text{CW}[tvu]}{3}+\frac{31\,\text{CW}[uuv]}{6}+\frac{13\,\text{CW}[uvv]}{6}\right]$$

True

$$\text{Print}\ /@\ \Big\{$$

$$0 \to \{\alpha,\ \beta\};$$

$$1 \to \left(\text{t0} = \frac{\text{J}[u,\ \alpha+\epsilon\,\beta]-\text{J}[u,\ \alpha]}{\epsilon}\right),$$

$$2 \to \left(\text{t1} = \int_0^1 (\text{div}[u,\ \beta\ //\ \text{RC}[u,\ s\,\alpha]]\ //\ \text{CC}[u,\ -s\,\alpha])\,\text{d}s\right),$$

$$3 \to \left(\text{t2} = \int_0^1 \left(\text{div}\left[u,\ \beta\ //\ \text{adSeries}\left[\frac{1-e^{-s\,\text{ad}}}{\text{ad}},\ \alpha\right]\ //\ \text{RC}[u,\ s\,\alpha]\ //\ \text{ad}[u,\ \alpha\ //\ \text{RC}[u,\ s\,\alpha]]\right]\ //\ \text{CC}[u,\ -s\,\alpha]\right)\text{d}s\right),$$

$$4 \to \left(\text{t3} = \int_0^1 \left(\text{div}\left[u,\ \beta\ //\ \text{adSeries}\left[\frac{1-e^{-s\,\text{ad}}}{\text{ad}},\ \alpha\right]\ //\ \text{RC}[u,\ s\,\alpha]\right]\ //\ \text{ad}[u,\ \alpha\ //\ \text{RC}[u,\ s\,\alpha]]\ //\ \text{CC}[u,\ -s\,\alpha]\right)\text{d}s\right),$$

$$5 \to \left(\text{t4} = \int_0^1 \left(\text{div}\left[u,\ \text{b}\left[\alpha,\ \beta\ //\ \text{adSeries}\left[\frac{1-e^{-s\,\text{ad}}}{\text{ad}},\ \alpha\right]\right]\ //\ \text{RC}[u,\ s\,\alpha]\right]\ //\ \text{CC}[u,\ -s\,\alpha]\right)\text{d}s\right),$$

$$\text{t0} \equiv \text{t1}+\text{t2}-\text{t3}-\text{t4}$$

$$\Big\};$$

$$1 \to \text{CWS}\left[2\,\text{CW}[u],\ -\frac{3\,\text{CW}[uv]}{2},\ -\frac{\text{CW}[ttu]}{6}+\frac{\text{CW}[tuu]}{2}+\frac{8\,\text{CW}[tuv]}{3}+\frac{2\,\text{CW}[tvu]}{3}+\frac{\text{CW}[uuv]}{6}-\frac{11\,\text{CW}[uvv]}{3}\right]$$

$$2 \to \text{CWS}\left[2\,\text{CW}[u],\ \frac{\text{CW}[uv]}{2},\ -\frac{\text{CW}[ttu]}{6}-\frac{\text{CW}[tuu]}{6}+\frac{7\,\text{CW}[tuv]}{3}-\frac{5\,\text{CW}[tvu]}{3}+\frac{5\,\text{CW}[uuv]}{3}\right]$$

$$3 \to \text{CWS}\left[0,\ -2\,\text{CW}[tu]+2\,\text{CW}[uv],\ -\frac{2\,\text{CW}[ttu]}{3}-\frac{8\,\text{CW}[tuu]}{3}+2\,\text{CW}[tuv]+\frac{\text{CW}[tvu]}{6}+\frac{11\,\text{CW}[uuv]}{3}-\frac{3\,\text{CW}[uvv]}{2}\right]$$

$$4 \to \text{CWS}\left[0,\ 0,\ \frac{5\,\text{CW}[tuv]}{6}-\frac{5\,\text{CW}[tvu]}{6}\right]$$

$$5 \to \text{CWS}\left[0,\ -2\,\text{CW}[tu]+4\,\text{CW}[uv],\ -\frac{2\,\text{CW}[ttu]}{3}-\frac{10\,\text{CW}[tuu]}{3}+\frac{5\,\text{CW}[tuv]}{6}-\frac{4\,\text{CW}[tvu]}{3}+\frac{31\,\text{CW}[uuv]}{6}+\frac{13\,\text{CW}[uvv]}{6}\right]$$

True

$$\texttt{Print /@}\ \Big\{$$

$$0 \to \{\alpha,\ \beta\};$$

$$1 \to \left(\text{t0} = \frac{\text{J}[u,\ \alpha+\epsilon\,\beta]-\text{J}[u,\ \alpha]}{\epsilon}\right),$$

$$2 \to \left(\text{t1} = \int_0^1 (\text{div}[u,\ \beta\ //\ \text{RC}[u,\ s\,\alpha]]\ //\ \text{CC}[u,\ -s\,\alpha])\,\text{d}s\right),$$

$$3 \to \left(\text{t2} = \int_0^1 \left(\text{div}\left[u,\ \beta\ //\ \text{adSeries}\left[\frac{1-e^{-s\,\text{ad}}}{\text{ad}},\ \alpha\right]\ //\ \text{RC}[u,\ s\,\alpha]\ //\ \text{ad}[u,\ \alpha\ //\ \text{RC}[u,\ s\,\alpha]]\right]\ //\ \text{CC}[u,\ -s\,\alpha]\right)\text{d}s\right),$$

$$4 \to \left(\text{t3} = \int_0^1 \left(\text{div}\left[u,\ \beta\ //\ \text{adSeries}\left[\frac{1-e^{-s\,\text{ad}}}{\text{ad}},\ \alpha\right]\ //\ \text{RC}[u,\ s\,\alpha]\right]\ //\ \text{ad}[u,\ \alpha\ //\ \text{RC}[u,\ s\,\alpha]]\ //\ \text{CC}[u,\ -s\,\alpha]\right)\text{d}s\right),$$

$$5 \to \left(\text{t4} = \int_0^1 \left(\text{div}\left[u,\ \beta\ //\ \text{adSeries}\left[1-e^{-s\,\text{ad}},\ \alpha\right]\ //\ \text{RC}[u,\ s\,\alpha]\right]\ //\ \text{CC}[u,\ -s\,\alpha]\right)\text{d}s\right),$$

$$\text{t0} \equiv \text{t1}+\text{t2}-\text{t3}-\text{t4}$$

$$\Big\};$$

$$1 \to \text{CWS}\left[2\,\text{CW}[u],\ -\frac{3\,\text{CW}[uv]}{2},\ -\frac{\text{CW}[ttu]}{6}+\frac{\text{CW}[tuu]}{2}+\frac{8\,\text{CW}[tuv]}{3}+\frac{2\,\text{CW}[tvu]}{3}+\frac{\text{CW}[uuv]}{6}-\frac{11\,\text{CW}[uvv]}{3}\right]$$

$$2 \to \text{CWS}\left[2\,\text{CW}[u],\ \frac{\text{CW}[uv]}{2},\ -\frac{\text{CW}[ttu]}{6}-\frac{\text{CW}[tuu]}{6}+\frac{7\,\text{CW}[tuv]}{3}-\frac{5\,\text{CW}[tvu]}{3}+\frac{5\,\text{CW}[uuv]}{3}\right]$$

$$3 \to \text{CWS}\left[0,\ -2\,\text{CW}[tu]+2\,\text{CW}[uv],\ -\frac{2\,\text{CW}[ttu]}{3}-\frac{8\,\text{CW}[tuu]}{3}+2\,\text{CW}[tuv]+\frac{\text{CW}[tvu]}{6}+\frac{11\,\text{CW}[uuv]}{3}-\frac{3\,\text{CW}[uvv]}{2}\right]$$

$$4 \to \text{CWS}\left[0,\ 0,\ \frac{5\,\text{CW}[tuv]}{6}-\frac{5\,\text{CW}[tvu]}{6}\right]$$

$$5 \to \text{CWS}\left[0,\ -2\,\text{CW}[tu]+4\,\text{CW}[uv],\ -\frac{2\,\text{CW}[ttu]}{3}-\frac{10\,\text{CW}[tuu]}{3}+\frac{5\,\text{CW}[tuv]}{6}-\frac{4\,\text{CW}[tvu]}{3}+\frac{31\,\text{CW}[uuv]}{6}+\frac{13\,\text{CW}[uvv]}{6}\right]$$

True

$$\texttt{Print /@}\ \Big\{$$

$$\texttt{0} \to \{\alpha,\ \beta\};$$

$$\texttt{1} \to \left(\texttt{t0} = \frac{\texttt{J[u, }\alpha + \epsilon\,\beta\texttt{] - J[u, }\alpha\texttt{]}}{\epsilon}\right),$$

$$\texttt{2} \to \left(\texttt{t1} = \int_0^1 \left(\texttt{div}\left[\texttt{u, }\beta\ \texttt{//}\ \texttt{adSeries}\left[\frac{1-e^{-s\,\text{ad}}}{\text{ad}},\ \alpha\right]\ \texttt{//}\ \texttt{RC[u, s }\alpha\texttt{]}\ \texttt{//}\ \texttt{ad[u, }\alpha\ \texttt{//}\ \texttt{RC[u, s }\alpha\texttt{]]}\right]\ \texttt{//}\ \texttt{CC[u, -s }\alpha\texttt{]}\right) \text{d}s\right),$$

$$\texttt{3} \to \left(\texttt{t2} = \int_0^1 \left(\texttt{div}\left[\texttt{u, }\beta\ \texttt{//}\ \texttt{adSeries}\left[\frac{1-e^{-s\,\text{ad}}}{\text{ad}},\ \alpha\right]\ \texttt{//}\ \texttt{RC[u, s }\alpha\texttt{]}\right]\ \texttt{//}\ \texttt{ad[u, }\alpha\ \texttt{//}\ \texttt{RC[u, s }\alpha\texttt{]]}\ \texttt{//}\ \texttt{CC[u, -s }\alpha\texttt{]}\right) \text{d}s\right),$$

$$\texttt{4} \to \left(\texttt{t3} = \int_0^1 \left(\texttt{div}\left[\texttt{u, }\beta\ \texttt{//}\ \texttt{adSeries}\left[e^{-s\,\text{ad}},\ \alpha\right]\ \texttt{//}\ \texttt{RC[u, s }\alpha\texttt{]}\right]\ \texttt{//}\ \texttt{CC[u, -s }\alpha\texttt{]}\right) \text{d}s\right),$$

$$\texttt{t0} \equiv \texttt{t1} - \texttt{t2} + \texttt{t3}$$

$$\Big\};$$

$$1 \to \text{CWS}\left[2\,\text{CW}[u],\ -\frac{3\,\text{CW}[uv]}{2},\ -\frac{\text{CW}[ttu]}{6}+\frac{\text{CW}[tuu]}{2}+\frac{8\,\text{CW}[tuv]}{3}+\frac{2\,\text{CW}[tvu]}{3}+\frac{\text{CW}[uuv]}{6}-\frac{11\,\text{CW}[uvv]}{3}\right]$$

$$2 \to \text{CWS}\left[0,\ -2\,\text{CW}[tu]+2\,\text{CW}[uv],\ -\frac{2\,\text{CW}[ttu]}{3}-\frac{8\,\text{CW}[tuu]}{3}+2\,\text{CW}[tuv]+\frac{\text{CW}[tvu]}{6}+\frac{11\,\text{CW}[uuv]}{3}-\frac{3\,\text{CW}[uvv]}{2}\right]$$

$$3 \to \text{CWS}\left[0,\ 0,\ \frac{5\,\text{CW}[tuv]}{6}-\frac{5\,\text{CW}[tvu]}{6}\right]$$

$$4 \to \text{CWS}\left[2\,\text{CW}[u],\ 2\,\text{CW}[tu]-\frac{7\,\text{CW}[uv]}{2},\ \frac{\text{CW}[ttu]}{2}+\frac{19\,\text{CW}[tuu]}{6}+\frac{3\,\text{CW}[tuv]}{2}-\frac{\text{CW}[tvu]}{3}-\frac{7\,\text{CW}[uuv]}{2}-\frac{13\,\text{CW}[uvv]}{6}\right]$$

True

```
Print /@ {
   0 → {α, β};
   1 → (t0 = (J[u, α + ϵ β] - J[u, α])/ϵ),
   2 → (t1 = ∫_0^1 (div[u, β // Ad[-s α] // RC[u, s α]] // CC[u, -s α]) ds),
   3 → (t2 = ∫_0^1 (div[u, β // adSeries[(1 - e^(-s ad))/ad, α] // RC[u, s α] //
           ad[u, α // RC[u, s α]]] // CC[u, -s α]) ds),
   4 → (t3 = ∫_0^1 (div[u, β // adSeries[(1 - e^(-s ad))/ad, α] // RC[u, s α]] //
         ad[u, α // RC[u, s α]] // CC[u, -s α]) ds),
   t0 ≡ t1 + t2 - t3
  };
```

$$1 \to \text{CWS}\left[2\,\text{CW}[u],\ -\frac{3\,\text{CW}[uv]}{2},\ -\frac{\text{CW}[ttu]}{6}+\frac{\text{CW}[tuu]}{2}+\frac{8\,\text{CW}[tuv]}{3}+\frac{2\,\text{CW}[tvu]}{3}+\frac{\text{CW}[uuv]}{6}-\frac{11\,\text{CW}[uvv]}{3}\right]$$

$$2 \to \text{CWS}\left[2\,\text{CW}[u],\ 2\,\text{CW}[tu]-\frac{7\,\text{CW}[uv]}{2},\ \frac{\text{CW}[ttu]}{2}+\frac{19\,\text{CW}[tuu]}{6}+\frac{3\,\text{CW}[tuv]}{2}-\frac{\text{CW}[tvu]}{3}-\frac{7\,\text{CW}[uuv]}{2}-\frac{13\,\text{CW}[uvv]}{6}\right]$$

$$3 \to \text{CWS}\left[0,\ -2\,\text{CW}[tu]+2\,\text{CW}[uv],\ -\frac{2\,\text{CW}[ttu]}{3}-\frac{8\,\text{CW}[tuu]}{3}+2\,\text{CW}[tuv]+\frac{\text{CW}[tvu]}{6}+\frac{11\,\text{CW}[uuv]}{3}-\frac{3\,\text{CW}[uvv]}{2}\right]$$

$$4 \to \text{CWS}\left[0,\ 0,\ \frac{5\,\text{CW}[tuv]}{6}-\frac{5\,\text{CW}[tvu]}{6}\right]$$

True

```
Print /@ {
    0 → {α, β};
    1 → (t0 = (J[u, α + ϵ β] - J[u, α])/ϵ),
    2 → (t1 = div[u, β // adSeries[(1 - e^-ad)/ad, α] // RC[u, α]] // CC[u, -α]),
    t0 ≡ t1
  };
```

$$1 \to \mathrm{CWS}\left[\mathrm{CW}[u],\ -\frac{5\,\mathrm{CW}[tu]}{2}+\frac{\mathrm{CW}[uv]}{2},\ \frac{\mathrm{CW}[ttu]}{6}+\mathrm{CW}[tuu]-2\,\mathrm{CW}[tuv]-\frac{\mathrm{CW}[tvu]}{4}-\frac{11\,\mathrm{CW}[uuv]}{4}-\frac{4\,\mathrm{CW}[uvv]}{3}\right]$$

$$2 \to \mathrm{CWS}\left[\mathrm{CW}[u],\ -\frac{5\,\mathrm{CW}[tu]}{2}+\frac{\mathrm{CW}[uv]}{2},\ \frac{\mathrm{CW}[ttu]}{6}+\mathrm{CW}[tuu]-2\,\mathrm{CW}[tuv]-\frac{\mathrm{CW}[tvu]}{4}-\frac{11\,\mathrm{CW}[uuv]}{4}-\frac{4\,\mathrm{CW}[uvv]}{3}\right]$$

True