Cheat Sheet β

The original β-calculus: With $\epsilon := 1+\alpha$, $\langle\alpha\rangle := \sum_v \alpha_v$, and $\langle\gamma\rangle := \sum_{v\neq u}\gamma_v$,

$$\begin{array}{c|c}\omega_1 & H_1\\ \hline T_1 & A_1\end{array} * \begin{array}{c|c}\omega_2 & H_2\\ \hline T_2 & A_2\end{array} \underset{\beta}{=} \begin{array}{c|cc}\omega_1\omega_2 & H_1 & H_2\\ \hline T_1 & A_1 & 0\\ T_2 & 0 & A_2\end{array}$$

$$\begin{array}{c|c}\omega & H\\ \hline u & \alpha\\ v & \beta\\ T & \gamma\end{array} \xrightarrow[\beta]{tm^{uv}_w} \begin{array}{c|c}\omega & H\\ \hline w & (\alpha+\beta) \mathbin{/\!\!/} (\begin{smallmatrix}u,v\\ \to w\end{smallmatrix})\\ T & \gamma\end{array}$$

$$R^\pm_{ux} \underset{\beta}{=} \begin{array}{c|c}1 & x\\ \hline u & t_u^{\pm1}-1\end{array}$$

$$\begin{array}{c|ccc}\omega & x & y & H\\ \hline T & \alpha & \beta & \gamma\end{array} \xrightarrow[\beta]{hm^{xy}_z} \begin{array}{c|cc}\omega & z & H\\ \hline T & \alpha+\beta+\langle\alpha\rangle\beta & \gamma\end{array}$$

$$\begin{array}{c|cc}\omega & x & H\\ \hline u & \alpha & \beta\\ T & \gamma & \delta\end{array} \xrightarrow[\beta]{sw^{ux}_{th}} \begin{array}{c|cc}\omega\epsilon & x & H\\ \hline u & \alpha(1+\langle\gamma\rangle/\epsilon) & \beta(1+\langle\gamma\rangle/\epsilon)\\ T & \gamma/\epsilon & \delta-\gamma\beta/\epsilon\end{array}$$

Constraints. • Column sums are monomials minus 1.

β-better calculus:

$$\begin{array}{c|c}\omega_1 & H_1\\ \hline T_1 & A_1\\ - & \sigma_1\end{array} * \begin{array}{c|c}\omega_2 & H_2\\ \hline T_2 & A_2\\ - & \sigma_2\end{array} \underset{\beta_b}{=} \begin{array}{c|cc}\omega_1\omega_2 & H_1 & H_2\\ \hline T_1 & \omega_2 A_1 & 0\\ T_2 & 0 & \omega_1 A_2\\ - & \sigma_1 & \sigma_2\end{array}$$

$$\begin{array}{c|c}\omega & H\\ \hline u & \alpha\\ v & \beta\\ T & \gamma\\ - & \sigma\end{array} \xrightarrow[\beta_b]{tm^{uv}_w} \begin{array}{c|c}\omega & H\\ \hline w & (\alpha+\beta) \mathbin{/\!\!/} (\begin{smallmatrix}u,v\\ \to w\end{smallmatrix})\\ T & \gamma\\ - & \sigma\end{array}$$

$$R^\pm_{ux} \underset{\beta_b}{=} \begin{array}{c|c}1 & x\\ \hline u & t_u^{\pm1}-1\\ - & t_u^{\pm1}\end{array}$$

$$\begin{array}{c|ccc}\omega & x & y & H\\ \hline T & \alpha & \beta & \gamma\\ - & \sigma_x & \sigma_y & \sigma\end{array} \xrightarrow[\beta_b]{hm^{xy}_z} \begin{array}{c|cc}\omega & z & H\\ \hline T & \alpha+\sigma_x\beta & \gamma\\ - & \sigma_x\sigma_y & \sigma\end{array}$$

$$\begin{array}{c|cc}\omega & x & H\\ \hline u & \alpha & \beta\\ T & \gamma & \delta\\ - & \sigma_x & \sigma\end{array} \xrightarrow[\beta_b]{sw^{ux}_{th}} \begin{array}{c|cc}\omega+\alpha & x & H\\ \hline u & \sigma_x\alpha & \sigma_x\beta\\ T & \gamma & \delta+\frac{\alpha\delta-\gamma\beta}{\omega} = \frac{(\omega+\alpha)\delta-\gamma\beta}{\omega}\\ - & \sigma_x & \sigma\end{array}$$

Constraints. • Sum of column x is $(\sigma_x - 1)w$. • Likely, $\omega^{k-1} \mid \Lambda^k A$.

From 2012-05/A Higher Minors Experiment:

$$\frac{1}{w^2}\begin{vmatrix}(w+\alpha)\delta_{11}-\gamma_1\beta_1 & (w+\alpha)\delta_{12}-\gamma_1\beta_2\\ (w+\alpha)\delta_{21}-\gamma_2\beta_1 & (w+\alpha)\delta_{22}-\gamma_2\beta_2\end{vmatrix} =$$

$$\begin{array}{ccc}\alpha & \beta_1 & \beta_2\\ \gamma_1 & \delta_{11} & \delta_{12}\\ \gamma_2 & \delta_{21} & \delta_{22}\end{array}$$
starting A

$$\frac{1}{w^2}\left[-\gamma_1(w+\alpha)\begin{vmatrix}\beta_1 & \beta_2\\ \delta_{21} & \delta_{22}\end{vmatrix} - \gamma_2(w+\alpha)\begin{vmatrix}\delta_{11} & \delta_{12}\\ \beta_1 & \beta_2\end{vmatrix} + (w+\alpha)^2\begin{vmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{vmatrix}\right]$$

$$= \frac{w+\alpha}{w^2}\underbrace{\left[\underbrace{w\underbrace{\begin{vmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{vmatrix}}_{\text{divisible by } w}}_{\text{div by } w^2} + \underbrace{\begin{vmatrix}\alpha & \beta_1 & \beta_2\\ \gamma_1 & \delta_{11} & \delta_{12}\\ \gamma_2 & \delta_{21} & \delta_{22}\end{vmatrix}}_{\text{div by } w^2}\right]}_{\text{div by } w+\alpha.}$$

To do. • Consider a verification program. • Add dm formulas. • Add Burau calculus. • Add the conjugation relation. • Add the MVA formula