change to // notation throughout!

Cheat Sheet J

http://drorbn.net/AcademicPensieve/2013-03/
initiated 18/3/13; completed ?; modified 26/3/13, 4:01pm

With alphabet T and with $u,v,w\in T$, $\alpha,\beta,\gamma\in FL(T)$, $D\in\mathfrak{tder}(T)$, $g,h\in\exp(\mathfrak{tder}(T))=\mathrm{TAut}(T)$. Checkmarks (✓) as in `CheatSheetJ-Verification.nb`.

---

1. The definition of J: $$J_u(\gamma) := \int_0^1 ds\,\mathrm{div}_u(\gamma /\!/ RC_u^{s\gamma}) /\!/ C_u^{-s\gamma}$$
2. ✓ The t equation (desired): $$J_w(\gamma /\!/ tm^{uv}_w) /\!/ RC_w^{\gamma/\!/tm^{uv}_w} = J_u(\gamma) /\!/ tm^{uv}_w /\!/ RC_w^{\gamma/\!/tm^{uv}_w} + J_v(\gamma /\!/ RC_u^\gamma) /\!/ RC_v^{\gamma/\!/RC_u^\gamma} /\!/ tm^{uv}_w$$
3. ✓ The h equation (desired): $$J_u(\mathrm{bch}(\alpha,\beta)) = J_u(\alpha) + J_u(\beta /\!/ RC_u^\alpha) /\!/ C_u^{-\alpha}$$

---

4. RC equation t: $$tm^{uv}_w /\!/ RC_w^{\gamma/\!/tm^{uv}_w} = RC_u^\gamma /\!/ RC_v^{\gamma/\!/RC_u^\gamma} /\!/ tm^{uv}_w$$
5. RC equation h: $$RC_u^{\mathrm{bch}(\alpha,\beta)} = RC_u^\alpha /\!/ RC_u^{\beta/\!/RC_u^\alpha}$$
6. RCC equation div: $$\mathrm{div}_u(\alpha /\!/ RC_u^\gamma) /\!/ C_u^\gamma = ?$$
7. CRC equation div: $$\mathrm{div}_u(\alpha /\!/ C_u^\gamma) /\!/ RC_u^\gamma = ?$$

---

8. div property t: $$\mathrm{div}_w(\gamma /\!/ tm^{uv}_w) = (\mathrm{div}_u(\gamma) + \mathrm{div}_v(\gamma)) /\!/ tm^{uv}_w$$
9. ✓ div property h — the "cocycle condition": with $\mathrm{ad}_u\{\gamma\} := \mathrm{der}(u\to[\gamma,u])$, $$(\mathrm{div}_u\,\alpha) /\!/ \mathrm{ad}_u\{\beta\} - (\mathrm{div}_u\,\beta) /\!/ \mathrm{ad}_u\{\alpha\} = \mathrm{div}_u\left([\alpha,\beta] + \alpha /\!/ \mathrm{ad}_u\{\beta\} - \beta /\!/ \mathrm{ad}_u\{\alpha\}\right)$$
10. div of bch: $$\mathrm{div}_u(\mathrm{bch}(\alpha,\beta)) = ?$$

---

11. The definition of JA: $$JA_u(\gamma) := J_u(\gamma) /\!/ RC_u^\gamma$$
12. The ODE for JA: with $\gamma_s = \gamma /\!/ RC_u^{s\gamma}$, $$JA(0)=0,\quad \frac{dJA(s)}{ds} = JA(s) /\!/ \mathrm{ad}_u\{\gamma_s\} + \mathrm{div}_u\,\gamma_s,\quad JA(1) = JA_u(\gamma)$$

---

13. The relation with $\mathfrak{tder}$: $$e^{\mathrm{ad}_u\{\gamma\}} = C_u^{?} \text{ and } C_u^\gamma = e^{\mathrm{ad}_u\{?\}}$$
14. The definition of j (following A-T): $$j(e^D) = \int_0^1 ds\, e^{sD}(\mathrm{div}\,D) = \frac{e^D-1}{D}(\mathrm{div}\,D)$$
15. j's cocycle property: $$j(gh) = j(g) + g\cdot j(h)$$

---

16. The differential of exp: $$\delta e^\gamma = e^\gamma\cdot\left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) = \left(\frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma}\right)(\delta\gamma)\cdot e^\gamma$$
17. ✓ The differential of $\gamma=\mathrm{bch}(\alpha,\beta)$: $$\left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) = \left(e^{-\mathrm{ad}\,\beta}\frac{1-e^{-\mathrm{ad}\,\alpha}}{\mathrm{ad}\,\alpha}\right)(\delta\alpha) + \left(\frac{1-e^{-\mathrm{ad}\,\beta}}{\mathrm{ad}\,\beta}\right)(\delta\beta)$$
18. ✓ The differential of C: $$\delta C_u^\gamma = \mathrm{ad}_u\left\{\left(\frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) /\!/ RC_u^{-\gamma}\right\} /\!/ C_u^\gamma$$
19. ✓ The differential of RC: $$\delta RC_u^\gamma = RC_u^\gamma /\!/ \mathrm{ad}_u\left\{\left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) /\!/ RC_u^\gamma\right\}$$

---

20. ✓ The differential of J: $$\delta J_u(\gamma) = \int_0^1 ds\,\mathrm{div}_u\left(\delta\gamma /\!/ e^{-\mathrm{ad}\,s\gamma} /\!/ RC_u^{s\gamma}\right) /\!/ C_u^{-s\gamma}$$

(Proof: Pensieve/2013-03) $$+ \int_0^1 ds\,\mathrm{div}_u\left(\left(\frac{1-e^{-\mathrm{ad}\,s\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) /\!/ RC_u^{s\gamma} /\!/ \mathrm{ad}_u\{\gamma /\!/ RC_u^{s\gamma}\}\right) /\!/ C_u^{-s\gamma}$$
$$- \int_0^1 ds\,\mathrm{div}_u\left(\left(\frac{1-e^{-\mathrm{ad}\,s\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) /\!/ RC_u^{s\gamma}\right) /\!/ \mathrm{ad}_u\{\gamma /\!/ RC_u^{s\gamma}\} /\!/ C_u^{-s\gamma}$$

pursue further