Cheat Sheet β

http://drorbn.net/AcademicPensieve/2013-03/
initiated 24/3/13; completed ?; modified 25/3/13, 10:10am

The original β-calculus: With $\epsilon := 1+\alpha$, $\langle\alpha\rangle := \sum_v \alpha_v$, and $\langle\gamma\rangle := \sum_{v\neq u}\gamma_v$;

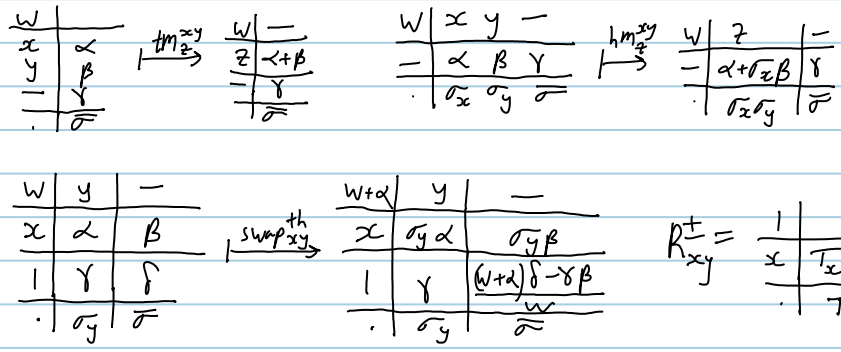
$$\begin{array}{c|c}\omega_1 & H_1\\ \hline T_1 & \alpha_1\end{array} * \begin{array}{c|c}\omega_2 & H_2\\ \hline T_2 & \alpha_2\end{array} = \begin{array}{c|cc}\omega_1\omega_2 & H_1 & H_2\\ \hline T_1 & \alpha_1 & 0\\ T_2 & 0 & \alpha_2\end{array} \qquad tm^{uv}_w : \begin{array}{c|c}\omega & \cdots\\ \hline u & \alpha\\ v & \beta\\ \vdots & \gamma\end{array} \mapsto \begin{array}{c|c}\omega & \cdots\\ \hline w & \alpha+\beta\\ \vdots & \gamma\end{array}$$

$$hm^{xy}_z : \begin{array}{c|ccc}\omega & x & y & \cdots\\ \hline \vdots & \alpha & \beta & \gamma\end{array} \mapsto \begin{array}{c|cc}\omega & z & \cdots\\ \hline \vdots & \alpha+\beta+\langle\alpha\rangle\beta & \gamma\end{array} \qquad tha^{ux} : \begin{array}{c|cc}\omega & x & \cdots\\ \hline u & \alpha & \beta\\ \vdots & \gamma & \delta\end{array} \mapsto \begin{array}{c|cc}\omega\epsilon & x & \cdots\\ \hline u & \alpha(1+\langle\gamma\rangle/\epsilon) & \beta(1+\langle\gamma\rangle/\epsilon)\\ \vdots & \gamma/\epsilon & \delta-\gamma\beta/\epsilon\end{array}$$

$$R^+_{ux} := \begin{array}{c|c}1 & x\\ \hline u & t_u-1\end{array} \qquad R^-_{ux} := \begin{array}{c|c}1 & x\\ \hline u & t_u^{-1}-1\end{array}.$$

$sw^{ux} \xrightarrow{\ \ } \beta$

From [beta-better cheat sheet]:



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{n = 3;
  b = B[ω, Sum[σ_j h[j], {j, n}], Sum[α_{10 i+j} t[i] h[j], {i, n}, {j, n}]],
  b // dm[1, 2, 1]
 } // ColumnForm
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from 2012-04/bbCalculus.nb

$$\begin{pmatrix} \omega & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} \\ 1+\Sigma/\omega & \sigma_1 & \sigma_2 & \sigma_3 \end{pmatrix}$$

$$\begin{pmatrix} \omega+\alpha_{12} & h[1] & h[3] \\ t[1] & \frac{(\omega+\alpha_{12})\,\alpha_{21}+\alpha_{22}\,(-\alpha_{11}+\omega\,\sigma_1)}{\omega} + (\alpha_{11}+\alpha_{12}\,\sigma_1)\,\sigma_2 & \frac{(\omega+\alpha_{12})\,\alpha_{23}+\alpha_{13}\,(-\alpha_{22}+\omega\,\sigma_2)}{\omega} \\ t[3] & \frac{(\omega+\alpha_{12})\,\alpha_{31}+\alpha_{32}\,(-\alpha_{11}+\omega\,\sigma_1)}{\omega} & \frac{-\alpha_{13}\,\alpha_{32}+(\omega+\alpha_{12})\,\alpha_{33}}{\omega} \\ 1+\Sigma/\omega & \sigma_1\,\sigma_2 & \sigma_3 \end{pmatrix}$$

$$\frac{\alpha(\omega+\alpha)\delta-\alpha\gamma\beta}{\omega}-\gamma\beta=\frac{\alpha(\omega+\alpha)\delta-(\omega+\alpha)\gamma\beta}{\omega}$$

$$=\frac{\omega+\alpha}{\omega}(\alpha\delta-\gamma\beta)$$