Calaque on "From associators to the Kashiwara-Vergne conjecture I (after Alekseev-Torossian)"

http://downloads.sms.cam.ac.uk/1432340/1432344.webm

March-10-13
12:20 PM

Motivation for the KV problem:

$\mathfrak{g}$: f.d. Lie algebra

$S(\mathfrak{g}) = \mathcal{O}_{\mathfrak{g}^*}$ is a Poisson algebra

$\rightsquigarrow$ quantization $U(\mathfrak{g})$

PBW isomorphism $\mathrm{sym}: S(\mathfrak{g})[t] \longrightarrow U(\mathfrak{g}[t], t[\,,\,])$

This gives a 1-parameter family of assoc. products on $S(\mathfrak{g})$:

$$m_t := \mathrm{sym}^{-1}(\mathrm{sym}(\#_1)\cdot \mathrm{sym}(\#_2))$$

m_0 is the original product, all m_t's are equiv. for $t \neq 0$.

<u>Thm</u> (Duflo) the composition 5:35

$$S(\mathfrak{g})^{\mathfrak{g}} \xrightarrow{\partial} S(\mathfrak{g})^{\mathfrak{g}} \xrightarrow{\mathrm{sym}} U(\mathfrak{g})^{\mathfrak{g}} = Z(U(\mathfrak{g}))$$

is an isomorphism, where

$\partial = \det(f(\mathrm{ad}\, x))$ where $f(x) = \left(\frac{1-e^{-x}}{x}\right)^{1/2}$

so $\partial \in \hat{S}(\mathfrak{g}^*)^{\mathfrak{g}}$ acts on $S(\mathfrak{g})^{\mathfrak{g}}$ as "differential operators"

KV strategy for proving Duflo's Thm

Prove that $\exists \beta : \mathfrak{g} \oplus \mathfrak{g} \longrightarrow \mathfrak{g} \oplus \mathfrak{g}$

$\mathfrak{g}$-equivariant, formal, pointed $(0 \mapsto 0)$

$\left(\text{i.e., } \beta \in \left(\hat{S}(\mathfrak{g}^* \oplus \mathfrak{g}^*) \otimes (\mathfrak{g} \oplus \mathfrak{g}) \right)^{\mathfrak{g}} \right)$

s.t.

$$\frac{d\tilde{m}_t}{dt} = -\tilde{m}_t \circ \tilde{\beta} \quad \text{where}$$

$\beta_t = t^{-1} \beta(t -)$, $\tilde{\beta}$ is the action of β_t on functions

$\tilde{m}_t = m_t \circ K_t$ where K = rescaling of

$$K = \frac{\partial(x)\partial(y)}{\partial(bch(x,y))}$$

22:44

I chose to skip the "rationale" for KV, until minute ...

... at the end, need to solve the equations

(1) $\frac{d\, bch_t}{dt} = \beta_t \cdot bch_t$ where $\beta_t \in Lie_a \oplus Lie_b$

acts as $x \longmapsto [a, x]$

$y \longmapsto [b, y]$

(2) $\frac{dK_t}{dt} = \rho\, K$ (discussion postponed)

(2) $\frac{dK_t}{dt} = \beta_t K_t$ (discussion postponed)

34:30