Pensieve header: Images for http://www.math.toronto.edu/~drorbn/Talks/Toronto-1303/.

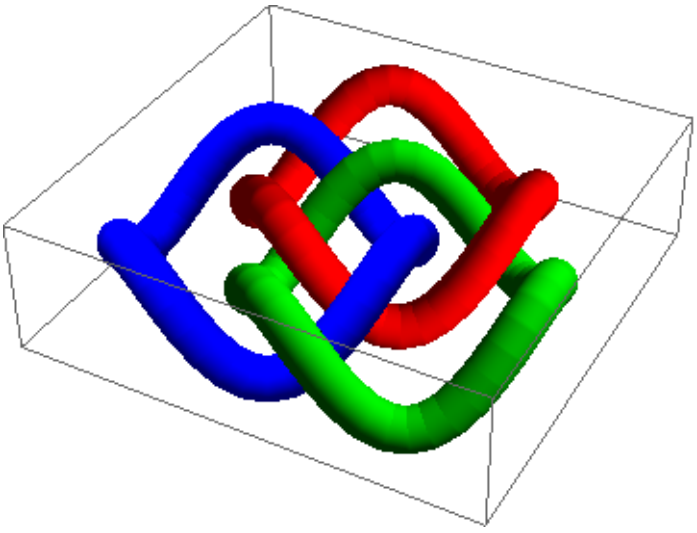
The Borromean Tangle

Formulas from http://paulbourke.net/geometry/borromean/

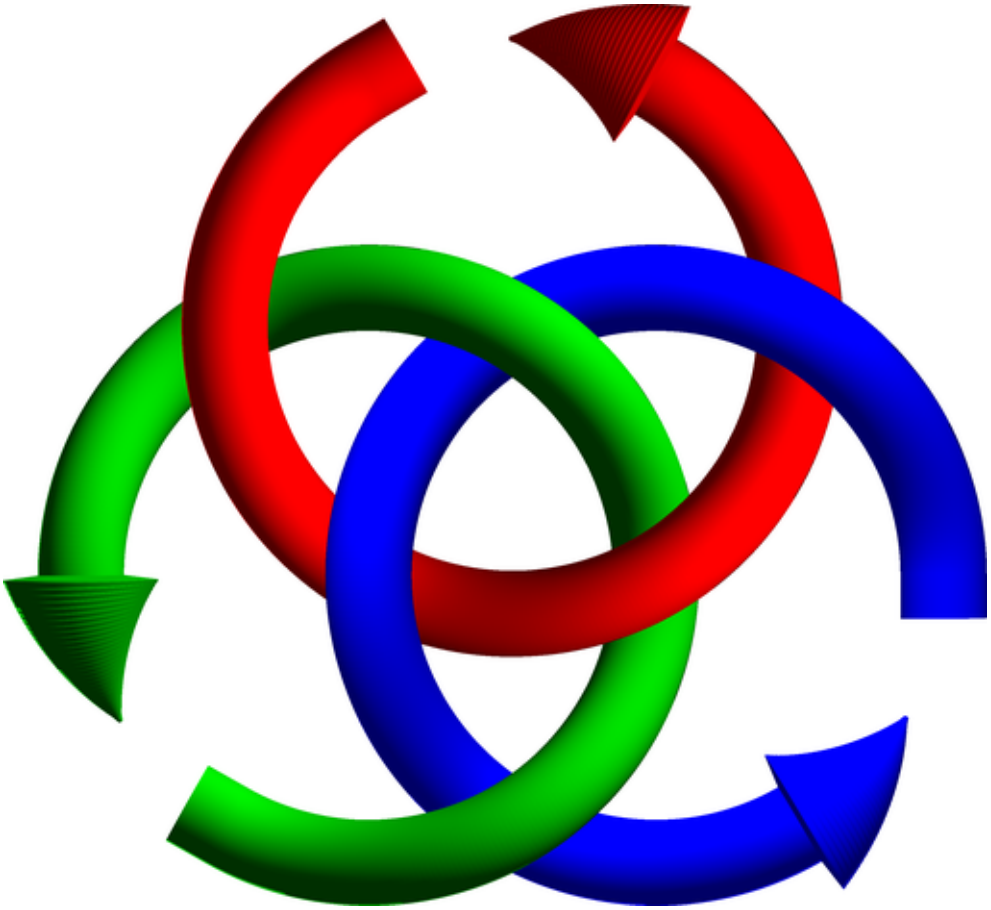
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-03"]
```

```
C:\drorbn\AcademicPensieve\2013-03
```

```
r = N[√3 / 3];
{s1, s2, s3} = Table[
    {
     {Cos[u], Sin[u] + r, Cos[3 u] / 3},
     {Cos[u] + 0.5, Sin[u] - r / 2, Cos[3 u] / 3},
     {Cos[u] - 0.5, Sin[u] - r / 2, Cos[3 u] / 3}
    },
    {u, 0., 2 π, 2 π / 48}
   ] // Transpose;
Rasterize[Graphics3D[{
   {Red, Tube[s1, 0.15]},
   {Green, Tube[s2, 0.15]},
   {Blue, Tube[s3, 0.15]}
  }]]
```



```
SetOptions[Rasterize, {RasterSize → 800, ImageSize → 800}];
rr = N[√3 / 3]; du = π / 192.;
s1 = Table[{Cos[u], Sin[u] + rr, -Cos[3 u] / 6}, {u, 4 π / 6, 15 π / 6, du}];
s2 = Table[{Cos[u] + 0.5, Sin[u] - rr / 2, -Cos[3 u] / 6}, {u, 12 π / 6, 23 π / 6, du}];
s3 = Table[{Cos[u] - 0.5, Sin[u] - rr / 2, -Cos[3 u] / 6}, {u, 8 π / 6, 19 π / 6, du}];
L = Length[s1];
rs = Table[Which[
     k > L / 11, 0.15,
     k ≤ L / 11, 3 k / L
    ], {k, L, 1, -1}];
gg = Graphics3D[{CapForm["Square"],
     {Red, Tube[s1, rs]},
     {Blue, Tube[s2, rs]},
     {Green, Tube[s3, rs]}
    },
    Boxed → False, ViewPoint → {0, 0, Infinity}];
MakeImage["BorromeanTangle", gg]
```



Trees

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-03"];
<< FreeLie.m
```

```
trees = LS[0, ⟨"bg"⟩, ⟨"bbg"⟩/2 + ⟨"bgg"⟩/2 + ⟨"bgr"⟩,
   ⟨"bbbg"⟩/6 + ⟨"bbgg"⟩/4 + ⟨"bbgr"⟩/2 + ⟨"bggg"⟩/6 + ⟨"bggr"⟩/2 + ⟨"bgrr"⟩/2,
   ⟨"bbbbg"⟩/24 + ⟨"bbbgg"⟩/12 + ⟨"bbbgr"⟩/6 - ⟨"bbgbg"⟩/12 + ⟨"bbggg"⟩/12 +
    ⟨"bbggr"⟩/4 - ⟨"bbgrg"⟩ + ⟨"bbgrr"⟩/4 - 2 ⟨"bbrgg"⟩ + ⟨"bgbgr"⟩/2 - ⟨"bgbrg"⟩ +
    ⟨"bgggg"⟩/24 + ⟨"bgggr"⟩/6 + ⟨"bggrr"⟩/4 - ⟨"bgrgr"⟩/2 + ⟨"bgrrr"⟩/6];
```

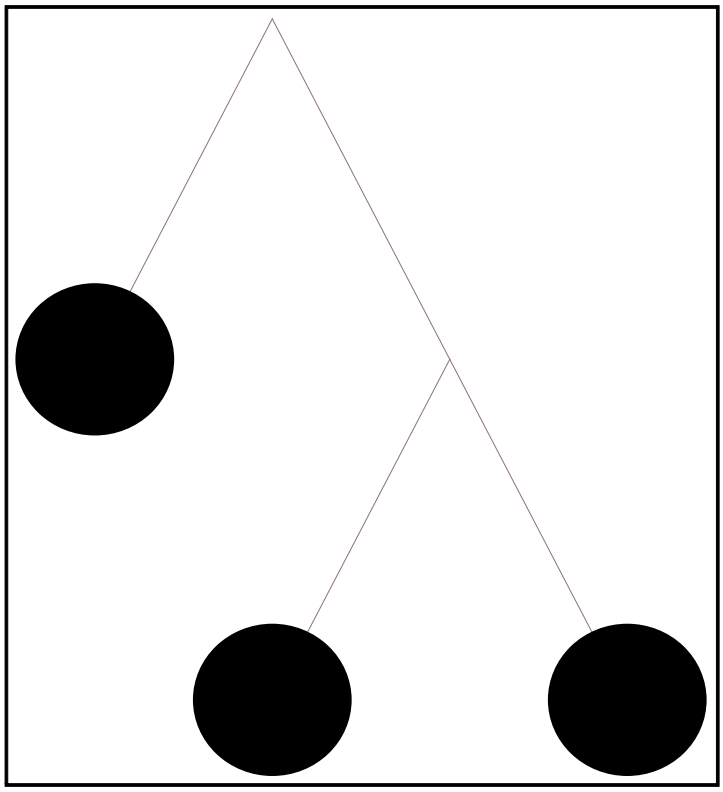
```
t1 = Series[
   (List @@ trees //. w_LW :> B @@ Reverse[LyndonFactorization[w]] /. B[s_] :> s /.
      t_B :> Tree[t]).ħ^Range[Length[trees]],
   {ħ, 0, Length[trees]}
  ] /. {"r" → r, "g" → g, "b" → b}
```

$$\begin{aligned}
&\text{Tree}[B[g, b]]\,\hbar^2 + \\
&\frac{1}{2}(\text{Tree}[B[g, B[g, b]]] + 2\,\text{Tree}[B[B[0.57735, g], b]] + \text{Tree}[B[B[g, b], b]])\,\hbar^3 + \\
&\frac{1}{12}(2\,\text{Tree}[B[g, B[g, B[g, b]]]] + 6\,\text{Tree}[B[B[0.57735, B[0.57735, g]], b]] + \\
&\quad 3\,\text{Tree}[B[B[g, B[g, b]], b]] + 6\,\text{Tree}[B[B[B[0.57735, g], b], b]] + \\
&\quad 6\,\text{Tree}[B[B[B[0.57735, g], g], b]] + 2\,\text{Tree}[B[B[B[g, b], b], b]])\,\hbar^4 + \\
&\frac{1}{24}(\text{Tree}[B[g, B[g, B[g, B[g, b]]]]] - 12\,\text{Tree}[B[B[0.57735, g], B[B[0.57735, g], b]]] + \\
&\quad 4\,\text{Tree}[B[B[0.57735, B[0.57735, B[0.57735, g]]], b]] - \\
&\quad 2\,\text{Tree}[B[B[g, b], B[B[g, b], b]]] - 24\,\text{Tree}[B[B[g, B[0.57735, b]], B[g, b]]] - \\
&\quad 48\,\text{Tree}[B[B[g, B[g, B[0.57735, b]]], b]] + 2\,\text{Tree}[B[B[g, B[g, B[g, b]]], b]] - 24 \\
&\quad \text{Tree}[B[B[g, B[B[0.57735, g], b]], b]] + 12\,\text{Tree}[B[B[B[0.57735, g], b], B[g, b]]] + \\
&\quad 6\,\text{Tree}[B[B[B[0.57735, B[0.57735, g]], b], b]] + \\
&\quad 6\,\text{Tree}[B[B[B[0.57735, B[0.57735, g]], g], b]] + 2\,\text{Tree}[B[B[B[g, B[g, b]], b], b]] + \\
&\quad 4\,\text{Tree}[B[B[B[B[0.57735, g], b], b], b]] + 6\,\text{Tree}[B[B[B[B[0.57735, g], g], b], b]] + \\
&\quad 4\,\text{Tree}[B[B[B[B[0.57735, g], g], g], b]] + \text{Tree}[B[B[B[B[g, b], b], b], b]])\,\hbar^5 + O[\hbar]^6
\end{aligned}$$

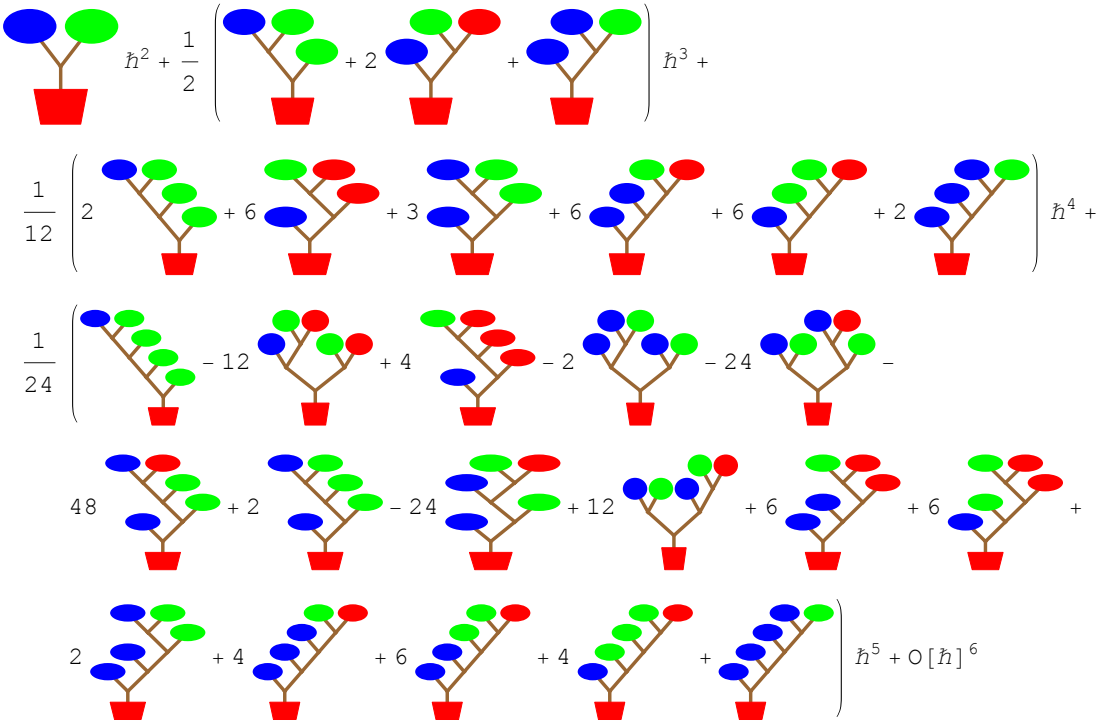
```
t = {x, {x, y}}
```

```
{x, {x, y}}
```

```
Framed[TreeForm[t,
  VertexRenderingFunction → (If[#2 === List, {},
      {
       Text[ToString[#2], #1],
       Disk[#1, 0.2]
      }
     ] &),
  PlotRangePadding → 0
 ]]
```



```
t1 /. t_Tree :> TreeForm[t,
   VertexRenderingFunction → (Switch[#2,
       Tree, {
        Red,
        Polygon[
         {{-0.4, 0.4} - #1, {0.4, 0.4} - #1, {0.3, -0.4} - #1, {-0.3, -0.4} - #1}]
       },
       B, {},
       _, {
        ReleaseHold[#2 /. {r → Red, g → Green, b → Blue}],
        Disk[-#1, 0.4]
       }
      ] &),
   EdgeRenderingFunction → ({
       Brown, Thickness[0.03],
       Line[-#]
      } &),
   PlotRangePadding → 0, ImageSize → 60, AspectRatio → 1
  ]
```



Wheels

```
data = CWS[0, 0, 2 CW["bgr"],
    CW["bbgr"] - CW["bgbr"] + CW["bggr"] - CW["bgrg"] + CW["bgrr"] - CW["brgr"],
    CW["bbbgr"]/3 - CW["bbgbr"]/2 + CW["bbggr"]/2 + CW["bbgrg"]/2 + CW["bbgrr"]/2 +
     CW["bbrbg"]/2 - 3 CW["bbrgr"]/2 + CW["bgbrr"]/2 - 3 CW["bggbr"]/2 +
     CW["bgggr"]/3 - CW["bggrg"]/2 + CW["bggrr"]/2 + CW["bgrgg"]/2 - 3 CW["bgrrg"]/2 +
     CW["bgrrr"]/3 + CW["brggr"]/2 - CW["brgrr"]/2 + CW["brrgr"]/2];
SetOptions[Rasterize, {RasterSize → 256, ImageSize → 256}];
Collect[
  Expand[(Plus @@ data)] /. CW[s_String] :> ħ^StringLength[s] Show[ImageCrop[PieChart3D[
        Table[1, {StringLength[s]}],
        ChartStyle :> (Characters[s] /. {"r" → Red, "g" → Green, "b" → Blue}),
        SectorOrigin → {{RandomReal[{0, 2 π}], "Counterclockwise"}, 1},
        ChartBaseStyle → EdgeForm[{Thickness[0.03], Black}],
        ChartElementFunction → "ProfileSector3D",
        ImagePadding → 0, ImageMargins → 0, PlotRangePadding → 0
       ]], ImageSize → 52],
  ħ, Factor] + O[ħ]^6
```