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ad[1] = x1;
ad[x1k] := Expand[(x1 + x2) x1k];
ad[p_Times] := Module[{j},
  j = Max[Cases[p, xi -> i, ∞]];
  Expand[p ∑i=1j+1 xi
];
ad[s_Plus] := ad /@ s

ad[1]
x1

1 // ad // ad // ad
x13 + 2 x12 x2 + x1 x22 + x1 x2 x3

1 // ad // ad // ad // ad
x14 + 3 x13 x2 + 3 x12 x22 + x1 x23 + 3 x12 x2 x3 + 2 x1 x22 x3 + x1 x2 x32 + x1 x2 x3 x4

sf[n_] := Module[{s = 1}, 1 + ∑k=1n Expand[ $\frac{s = ad[s]}{k!}$ ]]

sf[2]
1 + x1 +  $\frac{x_1^2}{2}$  +  $\frac{x_1 x_2}{2}$ 

sf[3]
1 + x1 +  $\frac{x_1^2}{2}$  +  $\frac{x_1^3}{6}$  +  $\frac{x_1 x_2}{2}$  +  $\frac{1}{3} x_1^2 x_2$  +  $\frac{1}{6} x_1 x_2^2$  +  $\frac{1}{6} x_1 x_2 x_3$ 

sf[5] /. xi -> ; i > 2 -> 0
1 + x1 +  $\frac{x_1^2}{2}$  +  $\frac{x_1^3}{6}$  +  $\frac{x_1^4}{24}$  +  $\frac{x_1^5}{120}$  +  $\frac{x_1 x_2}{2}$  +  $\frac{1}{3} x_1^2 x_2$  +  $\frac{1}{8} x_1^3 x_2$  +
 $\frac{1}{30} x_1^4 x_2$  +  $\frac{1}{6} x_1 x_2^2$  +  $\frac{1}{8} x_1^2 x_2^2$  +  $\frac{1}{20} x_1^3 x_2^2$  +  $\frac{1}{24} x_1 x_2^3$  +  $\frac{1}{30} x_1^2 x_2^3$  +  $\frac{1}{120} x_1 x_2^4$ 

f[x_] = x +  $\frac{x^2}{2}$  +  $\frac{x^3}{6}$ ;
Expand[1 + f[x1] +  $\frac{f[x_1] f[x_2]}{2}$ ]
1 + x1 +  $\frac{x_1^2}{2}$  +  $\frac{x_1^3}{6}$  +  $\frac{x_1 x_2}{2}$  +  $\frac{1}{4} x_1^2 x_2$  +  $\frac{1}{12} x_1^3 x_2$  +
 $\frac{1}{4} x_1 x_2^2$  +  $\frac{1}{8} x_1^2 x_2^2$  +  $\frac{1}{24} x_1^3 x_2^2$  +  $\frac{1}{12} x_1 x_2^3$  +  $\frac{1}{24} x_1^2 x_2^3$  +  $\frac{1}{72} x_1^3 x_2^3$ 

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