Pensieve header: Experiments for pensieve://Projects/KBH.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-03"];
<< FreeLie.m

$SeriesCompareDegree = 5;

Domain[f_List] := First /@ f;
f_ \ key_ := DeleteCases[f, key → _];
f_ \ keys_List := Fold[#1 \ #2 &, f, keys];
f1_List ≡ f2_List := Domain[f1] === Domain[f2] && (And @@ (
       ((# /. f1) ≡ (# /. f2)) & /@ Domain[f1]
     ));
(* LieDerivation[der_][f_List] := MapAt[LieDerivation[der], f, {All, 2}]; *)
LieMorphism[mor_][f_List] := MapAt[LieMorphism[mor], f, {All, 2}];
M /: M[λ1_, ω1_] ⋃ M[λ2_, ω2_] := M[λ1 ⋃ λ2, ω1 + ω2];
M[λ1_, ω1_] ≡ M[λ2_, ω2_] := (λ1 ≡ λ2) && (ω1 ≡ ω2);

tm[u_, v_, w_][λ_List] := λ // LieMorphism[⟨u⟩ → ⟨w⟩, ⟨v⟩ → ⟨w⟩];
tm[u_, v_, w_][M[λ_, ω_]] := LieMorphism[⟨u⟩ → ⟨w⟩, ⟨v⟩ → ⟨w⟩] /@ M[λ, ω];
hm[x_, y_, z_][λ_List] := Union[λ \ {x, y}, {z → BCH[x /. λ, y /. λ]}];
hm[x_, y_, z_][M[λ_, ω_]] := M[λ // hm[x, y, z], ω];
RC[u_, λx_LieSeries, ub_][ser_] := StableApply[
   LieMorphism[⟨u⟩ → Ad[λx][⟨ub⟩]],
   ser
  ];
RC[u_, λx_LieSeries][ser_] :=
  ser // RC[⟨u⟩, λx, ⟨"υ"⟩] // LieMorphism[⟨"υ"⟩ → ⟨u⟩];
J[u_, λx_] := Module[{s},
   IntegrateCWSeries[
    div[⟨u⟩, λx // RC[⟨u⟩, s λx]] // LieMorphism[⟨u⟩ → Ad[-s λx][⟨u⟩]],
    {s, 0, 1}
   ]
  ];
tha[u_, x_][λ_List] := MapAt[RC[⟨u⟩, x /. λ], λ, {All, 2}];
tha[u_, x_][M[λ_, ω_]] :=
  M[λ // tha[u, x], (ω + J[⟨u⟩, x /. λ]) // RC[⟨u⟩, x /. λ]];
dm[a_, b_, c_][μ_] := μ // tha[⟨a⟩, b] // tm[⟨a⟩, ⟨b⟩, ⟨c⟩] // hm[a, b, c];
R⁺[u_, x_] := M[{x → MakeLieSeries[⟨u⟩], u → MakeLieSeries[0]}, MakeCWSeries[0]];
R⁻[u_, x_] := M[{x → MakeLieSeries[-⟨u⟩], u → MakeLieSeries[0]}, MakeCWSeries[0]];
```

Testing t-action

```
$SeriesShowDegree = $SeriesCompareDegree = 5;
Print /@ {{u = ⟨"u"⟩, v = ⟨"v"⟩, w = ⟨"w"⟩, τ = ⟨"τ"⟩, t = ⟨"t"⟩};
   1 → (t1 = M[{
         x → MakeLieSeries[u + b[u, t]],
         y → MakeLieSeries[u + b[u, τ]]
        }, MakeCWSeries[0]]),
   t1 // tm[u, v, w],
   2 → (t2 = t1 // tm[u, v, w] // tha[w, x]),
   3 → (t3 = t1 // tha[u, x] // tha[v, x] // tm[u, v, w]),
   4 → (t2 ≡ t3)
  };
$SeriesShowDegree = 3;
```

$$1 \to M\left[\left\{x \to LS\left[u, -\overline{tu}, 0, 0, 0\right], y \to LS\left[u, \overline{u\tau}, 0, 0, 0\right]\right\}, CWS[0, 0, 0, 0, 0]\right]$$

$$M\left[\left\{x \to LS\left[w, -\overline{tw}, 0, 0, 0\right], y \to LS\left[w, \overline{w\tau}, 0, 0, 0\right]\right\}, CWS[0, 0, 0, 0, 0]\right]$$

$$2 \to M\left[\left\{x \to LS\left[w, -\overline{tw}, -\overline{\overline{tww}}, \overline{t\overline{\overline{tww}}w} - \frac{1}{2}\overline{\overline{\overline{tww}}ww}, \frac{3}{2}\overline{t\overline{\overline{tww}}ww} + \frac{3}{2}\overline{\overline{tw}\overline{tww}} - \frac{1}{6}\overline{\overline{\overline{tww}}www}\right],\right.$$

$$y \to LS\left[w, \overline{w\tau}, -\overline{\overline{tww}}, -\overline{t\overline{w\overline{w\tau}}} - 2\overline{t\overline{w\tau}w} - \frac{1}{2}\overline{\overline{tww}ww} - \overline{\overline{t\tau w}w},\right.$$

$$\left.\left.-\frac{1}{2}\overline{tw\overline{w\overline{w\tau}}} + \overline{t\overline{tww}w} + \frac{3}{2}\overline{\overline{tw}\overline{tww}} - \frac{3}{2}\overline{t\overline{w\overline{w\tau}}w} - \frac{3}{2}\overline{t\overline{w\tau}ww} - \frac{1}{6}\overline{\overline{tww}www} - \frac{1}{2}\overline{\overline{t\tau w}ww}\right]\right\},$$

$$CWS\left[CW[w], -CW[tw], \frac{CW[tww]}{2}, \frac{3\,CW[ttww]}{2} - 2\,CW[twtw] - \frac{CW[twww]}{6},\right.$$

$$\left.-\frac{5\,CW[ttwww]}{6} + \frac{4\,CW[twtww]}{3} + \frac{CW[twwww]}{24}\right]\Bigg]$$

$$3 \to M\left[\left\{x \to LS\left[w, -\overline{tw}, -\overline{\overline{tww}}, \overline{t\overline{\overline{tww}}w} - \frac{1}{2}\overline{\overline{\overline{tww}}ww}, \frac{3}{2}\overline{t\overline{\overline{tww}}ww} + \frac{3}{2}\overline{\overline{tw}\overline{tww}} - \frac{1}{6}\overline{\overline{\overline{tww}}www}\right],\right.$$

$$y \to LS\left[w, \overline{w\tau}, -\overline{\overline{tww}}, -\overline{t\overline{w\overline{w\tau}}} - 2\overline{t\overline{w\tau}w} - \frac{1}{2}\overline{\overline{tww}ww} - \overline{\overline{t\tau w}w},\right.$$

$$\left.\left.-\frac{1}{2}\overline{tw\overline{w\overline{w\tau}}} + \overline{t\overline{tww}w} + \frac{3}{2}\overline{\overline{tw}\overline{tww}} - \frac{3}{2}\overline{t\overline{w\overline{w\tau}}w} - \frac{3}{2}\overline{t\overline{w\tau}ww} - \frac{1}{6}\overline{\overline{tww}www} - \frac{1}{2}\overline{\overline{t\tau w}ww}\right]\right\},$$

$$CWS\left[CW[w], -CW[tw], \frac{CW[tww]}{2}, \frac{3\,CW[ttww]}{2} - 2\,CW[twtw] - \frac{CW[twww]}{6},\right.$$

$$\left.-\frac{5\,CW[ttwww]}{6} + \frac{4\,CW[twtww]}{3} + \frac{CW[twwww]}{24}\right]\Bigg]$$

$$4 \to True$$

The Borromean Tangle

```
μ0 = R⁻[r, 6] ⋃ R⁺[2, 4] ⋃ R⁻[g, 9] ⋃ R⁺[5, 7] ⋃ R⁻[b, 3] ⋃ R⁺[8, 1]
```

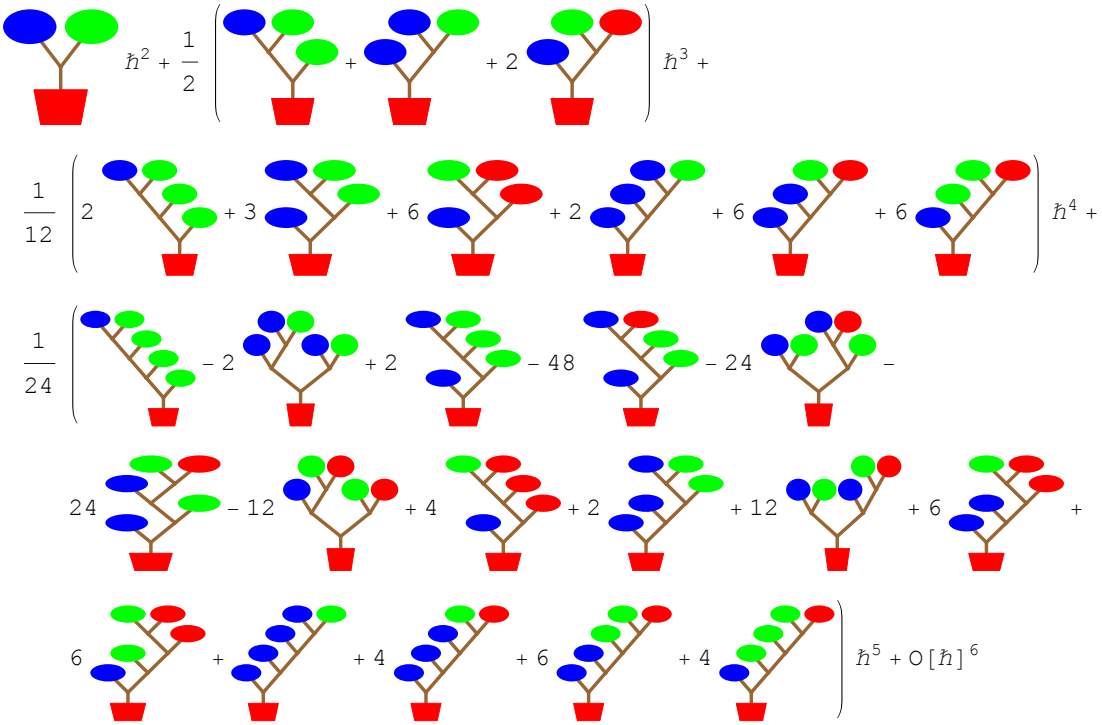
$$M[\{1 \to LS[8, 0, 0], 2 \to LS[0, 0, 0], 3 \to LS[-b, 0, 0], 4 \to LS[2, 0, 0],$$
$$5 \to LS[0, 0, 0], 6 \to LS[-r, 0, 0], 7 \to LS[5, 0, 0], 8 \to LS[0, 0, 0],$$
$$9 \to LS[-g, 0, 0], b \to LS[0, 0, 0], g \to LS[0, 0, 0], r \to LS[0, 0, 0]\}, CWS[0, 0, 0]]$$

```
Do[μ0 = μ0 // dm[r, k, r], {k, 1, 3}];
Do[μ0 = μ0 // dm[g, k, g], {k, 4, 6}];
Do[μ0 = μ0 // dm[b, k, b], {k, 7, 9}];
μ0
```

$$\mathrm{M}\Big[\Big\{\mathrm{b}\to \mathrm{LS}\Big[0,\ \overline{\mathrm{g\,r}},\ \frac{1}{2}\,\overline{\mathrm{g}\,\overline{\mathrm{g\,r}}}+\overline{\overline{\mathrm{b\,r}}\,\mathrm{g}}+\frac{1}{2}\,\overline{\overline{\mathrm{g\,r}}\,\mathrm{r}}\Big],\ \mathrm{g}\to \mathrm{LS}\Big[0,\ -\overline{\mathrm{b\,r}},\ \frac{1}{2}\,\overline{\mathrm{b}\,\overline{\mathrm{b\,r}}}-\overline{\mathrm{b}\,\overline{\mathrm{g\,r}}}-\overline{\overline{\mathrm{b\,r}}\,\mathrm{g}}+\frac{1}{2}\,\overline{\overline{\mathrm{b\,r}}\,\mathrm{r}}\Big],$$
$$\mathrm{r}\to \mathrm{LS}\Big[0,\ \overline{\mathrm{b\,g}},\ \frac{1}{2}\,\overline{\mathrm{b}\,\overline{\mathrm{b\,g}}}+\overline{\mathrm{b}\,\overline{\mathrm{g\,r}}}+\frac{1}{2}\,\overline{\overline{\mathrm{b\,g}}\,\mathrm{g}}\Big]\Big\},\ \mathrm{CWS}[0,\ 0,\ 2\,\mathrm{CW}[\mathrm{bgr}]]\Big]$$

Trees

```
trees = Table[(r /. First[μ0])@k, {k, 5}];
t1 = Series[(List@@trees //. w_LW :> B@@Reverse[LyndonFactorization[w]] /.
       B[s_] :> s /. t_B :> Tree[t]).ħ^Range[Length[trees]],
   {ħ, 0, Length[trees]}
  ] /. {"r" → r, "g" → g, "b" → b};
t1 /. t_Tree :> TreeForm[t,
   VertexRenderingFunction → (Switch[#2,
       Tree, {
        Red,
        Polygon[
         {{-0.4, 0.4} - #1, {0.4, 0.4} - #1, {0.3, -0.4} - #1, {-0.3, -0.4} - #1}]
       },
       B, {},
       _, {
        ReleaseHold[#2 /. {r → Red, g → Green, b → Blue}],
        Disk[-#1, 0.4]
       }
      ] &),
   EdgeRenderingFunction → ({
       Brown, Thickness[0.03],
       Line[-#]
      } &),
   PlotRangePadding → 0, ImageSize → 60, AspectRatio → 1
  ]
```



Wheels

```
n = 4;
wheels = Table[Last[μ0]@k, {k, n}];
SetOptions[Rasterize, {RasterSize → 256, ImageSize → 256}];
Collect[
  Expand[(Plus @@ wheels)] /.
   CW[s_String] :↦ ħ^StringLength[s] Show[ImageCrop[PieChart3D[
         Table[1, {StringLength[s]}],
         ChartStyle :↦ (Characters[s] /. {"r" → Red, "g" → Green, "b" → Blue}),
         SectorOrigin → {{RandomReal[{0, 2 π}], "Counterclockwise"}, 1},
         ChartBaseStyle → EdgeForm[{Thickness[0.03], Black}],
         ChartElementFunction → "ProfileSector3D",
         ImagePadding → 0, ImageMargins → 0, PlotRangePadding → 0
        ]], ImageSize → 52],
  ħ, Factor] + O[ħ]^(n+1)
```