Group cohomology

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1:33 PM

G – a group

M – a G-module

comes from the "invariants" functor.

$C^n(G, M)$: functions $G^n \longrightarrow M$

if $\varphi \in C^n(G,M)$, $d\varphi \in C^{n+1}(G, M)$ by

$$(d\varphi)(g_1 \ldots g_{n+1}) = g_1 \varphi(g_2 \ldots g_{n+1})$$
$$+ \sum_{i=1}^{n} (-1)^i \varphi(g_1 \ldots g_{i-1}, g_i \cdot g_{i+1}, \ldots g_{n+1})$$
$$+ (-1)^{n+1} \varphi(g_1 \ldots g_n)$$

If M is $\mathbb{K}$, this yield $H^*(K(G,1))$

in that case, $H^1 = \mathrm{Hom}(G, \mathbb{K})$

H^2 = "central extensions by $\mathbb{K}$".

What is the cup product?

I should see if this is related to Gauss diagram formulas.

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cup product: $\varphi \in C^n$, $\psi \in C^m$

$$(\varphi \cup \psi)(g_1, \ldots, g_{n+m}) = \sum_{\sigma} (-1)^{\sigma} \varphi(g_{\sigma 1} \ldots g_{\sigma n}) \psi(g_{\sigma(n+1)}, \ldots)$$

where $\sigma \in S_{n+m}$ is monotone on $1 \ldots n$ & on $(n+1) \ldots (n+m)$.

$$d(\varphi \cup \psi) = (d\varphi) \cup \psi \pm \varphi \cup (d\psi)$$