

## Random Stuff

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PJ[u_, λ_] := J[u, λ] // LieMorphism[u → Ad[-λ][u]];

$SeriesShowDegree = 3;
{u = <"u">, w = <"w">, λ0 = MakeLieSeries[u + w + b[u, w]]}
{<u>, <w>, LS[<u> + <w>, <uw>, 0]}

s λ0
LS[s <u> + s <w>, s <uw>, 0]

λ0 // CC[s λ0, u]

LS[<u> + <w>, <uw> - s <uw>, -s <uuw> +  $\frac{1}{2}$  s2 <uuw> - s <uww> +  $\frac{1}{2}$  s2 <uww>]

div[u, λ0 // CC[s λ0, u]]

CWS[CW[u], -CW[uw] + s CW[uw], s CW[uuw] -  $\frac{1}{2}$  s2 CW[uuw] - s CW[uww] +  $\frac{1}{2}$  s2 CW[uww]]

div[u, λ0 // CC[s λ0, u]] // LieMorphism[u → Ad[-s λ0][u]]

CWS[CW[u], -CW[uw] + s CW[uw], s CW[uuw] -  $\frac{1}{2}$  s2 CW[uuw] - s CW[uww] +  $\frac{1}{2}$  s2 CW[uww]]

t1 = IntegrateCWSeries[
  div[u, λ0 // CC[s λ0, u]] // LieMorphism[u → Ad[-s λ0][u]],
  {s, 0, 1}
]

CWS[CW[u], - $\frac{CW[uw]}{2}$ ,  $\frac{CW[uuw]}{3}$  -  $\frac{CW[uww]}{3}$ ]

J[u, λ0]

CWS[CW[u], - $\frac{CW[uw]}{2}$ ,  $\frac{CW[uuw]}{3}$  -  $\frac{CW[uww]}{3}$ ]

t2 = (J[u, λ0] // LieMorphism[u → Ad[-λ0][u]])

CWS[CW[u], - $\frac{CW[uw]}{2}$ ,  $\frac{CW[uuw]}{3}$  -  $\frac{CW[uww]}{3}$ ]

t1 ≡ t2
True

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{u = <"u">, w = <"w">, λ0 = MakeLieSeries[u + w + b[u, w]]};
{PJ[u, s λ0], P[u, s λ0]}

{CWS[s CW[u], -s CW[uw] + 1/2 s^2 CW[uw],
 1/2 s^2 CW[uuw] - 1/6 s^3 CW[uuw] - 1/2 s^2 CW[uww] + 1/6 s^3 CW[uww]], CWS[s CW[u],
-s CW[uw] + 1/2 s^2 CW[uw], 1/2 s^2 CW[uuw] - 1/6 s^3 CW[uuw] - 1/2 s^2 CW[uww] + 1/6 s^3 CW[uww]]}

div[u, λ0]
CWS[CW[u], -CW[uw], 0]

Table[
Coefficient[P[u, s λ0]@k, s] == div[u, λ0]@k,
{k, 8}
]
{True, True, True, True, True, True, True, True}

P[u, λ0] == PJ[u, λ0]
True

PJ[u, ε λ0 // CC[s λ0, u]]
CWS[ε CW[u], -ε CW[uw] + s ε CW[uw] + 1/2 ε^2 CW[uw],
s ε CW[uuw] - 1/2 s^2 ε CW[uuw] + 1/2 ε^2 CW[uuw] - 1/2 s ε^2 CW[uuw] - 1/6 ε^3 CW[uuw] -
s ε CW[uww] + 1/2 s^2 ε CW[uww] - 1/2 ε^2 CW[uww] + 1/2 s ε^2 CW[uww] + 1/6 ε^3 CW[uww]]

div[u, λ0 // CC[s λ0, u]]
CWS[CW[u], -CW[uw] + s CW[uw], s CW[uuw] - 1/2 s^2 CW[uuw] - s CW[uww] + 1/2 s^2 CW[uww]]

Table[
Coefficient[PJ[u, ε λ0 // CC[s λ0, u]]@k, ε] == div[u, λ0 // CC[s λ0, u]]@k,
{k, 6}
]
{True, True, True, True, True, True}

Table[
Coefficient[P[u, ε λ0 // CC[s λ0, u]]@k, ε] == div[u, λ0 // CC[s λ0, u]]@k,
{k, 6}
]
{True, True, True, True, True, True}

```

**PJ[u, s λ0]**

$$\text{CWS} \left[ s \text{ CW}[u], -s \text{ CW}[uw] + \frac{1}{2} s^2 \text{ CW}[uw], \right. \\ \left. \frac{1}{2} s^2 \text{ CW}[uuw] - \frac{1}{6} s^3 \text{ CW}[uuw] - \frac{1}{2} s^2 \text{ CW}[uww] + \frac{1}{6} s^3 \text{ CW}[uww] \right]$$

**div[u, λ0 // CC[s λ0, u]] //**

Even more random

**P[u, (s + ε) λ0]**

$$\text{CWS} \left[ s \text{ CW}[u] + \epsilon \text{ CW}[u], -s \text{ CW}[uw] + \frac{1}{2} s^2 \text{ CW}[uw] - \epsilon \text{ CW}[uw] + s \epsilon \text{ CW}[uw] + \frac{1}{2} \epsilon^2 \text{ CW}[uw], \right. \\ \frac{1}{2} s^2 \text{ CW}[uuw] - \frac{1}{6} s^3 \text{ CW}[uuw] + s \epsilon \text{ CW}[uuw] - \frac{1}{2} s^2 \epsilon \text{ CW}[uuw] + \frac{1}{2} \epsilon^2 \text{ CW}[uuw] - \\ \frac{1}{2} s \epsilon^2 \text{ CW}[uuw] - \frac{1}{6} \epsilon^3 \text{ CW}[uuw] - \frac{1}{2} s^2 \text{ CW}[uww] + \frac{1}{6} s^3 \text{ CW}[uww] - s \epsilon \text{ CW}[uww] + \\ \left. \frac{1}{2} s^2 \epsilon \text{ CW}[uww] - \frac{1}{2} \epsilon^2 \text{ CW}[uww] + \frac{1}{2} s \epsilon^2 \text{ CW}[uww] + \frac{1}{6} \epsilon^3 \text{ CW}[uww] \right]$$

**t1 = P[u, (s + ε) λ0] // CC[s λ0, u]**

$$\text{CWS} \left[ s \text{ CW}[u] + \epsilon \text{ CW}[u], -s \text{ CW}[uw] + \frac{1}{2} s^2 \text{ CW}[uw] - \epsilon \text{ CW}[uw] + s \epsilon \text{ CW}[uw] + \frac{1}{2} \epsilon^2 \text{ CW}[uw], \right. \\ \frac{1}{2} s^2 \text{ CW}[uuw] - \frac{1}{6} s^3 \text{ CW}[uuw] + s \epsilon \text{ CW}[uuw] - \frac{1}{2} s^2 \epsilon \text{ CW}[uuw] + \frac{1}{2} \epsilon^2 \text{ CW}[uuw] - \\ \frac{1}{2} s \epsilon^2 \text{ CW}[uuw] - \frac{1}{6} \epsilon^3 \text{ CW}[uuw] - \frac{1}{2} s^2 \text{ CW}[uww] + \frac{1}{6} s^3 \text{ CW}[uww] - s \epsilon \text{ CW}[uww] + \\ \left. \frac{1}{2} s^2 \epsilon \text{ CW}[uww] - \frac{1}{2} \epsilon^2 \text{ CW}[uww] + \frac{1}{2} s \epsilon^2 \text{ CW}[uww] + \frac{1}{6} \epsilon^3 \text{ CW}[uww] \right]$$

**t2 = P[u, s λ0] // CC[s λ0, u]**

$$\text{CWS} \left[ s \text{ CW}[u], -s \text{ CW}[uw] + \frac{1}{2} s^2 \text{ CW}[uw], \right. \\ \left. \frac{1}{2} s^2 \text{ CW}[uuw] - \frac{1}{6} s^3 \text{ CW}[uuw] - \frac{1}{2} s^2 \text{ CW}[uww] + \frac{1}{6} s^3 \text{ CW}[uww] \right]$$

**ε λ0 // CC[s λ0, u]**

$$\text{LS} \left[ \epsilon \langle u \rangle + \epsilon \langle w \rangle, \epsilon \langle uw \rangle - s \epsilon \langle uw \rangle, -s \epsilon \langle uww \rangle + \frac{1}{2} s^2 \epsilon \langle uww \rangle - s \epsilon \langle uww \rangle + \frac{1}{2} s^2 \epsilon \langle uww \rangle \right]$$

**P[u, ε λ0 // CC[s λ0, u]]**

$$\text{CWS} \left[ \epsilon \text{ CW}[u], -\epsilon \text{ CW}[uw] + s \epsilon \text{ CW}[uw] + \frac{1}{2} \epsilon^2 \text{ CW}[uw], \right. \\ s \epsilon \text{ CW}[uww] - \frac{1}{2} s^2 \epsilon \text{ CW}[uww] + \frac{1}{2} \epsilon^2 \text{ CW}[uww] - \frac{1}{2} s \epsilon^2 \text{ CW}[uww] - \frac{1}{6} \epsilon^3 \text{ CW}[uww] - \\ \left. s \epsilon \text{ CW}[uww] + \frac{1}{2} s^2 \epsilon \text{ CW}[uww] - \frac{1}{2} \epsilon^2 \text{ CW}[uww] + \frac{1}{2} s \epsilon^2 \text{ CW}[uww] + \frac{1}{6} \epsilon^3 \text{ CW}[uww] \right]$$

**t3 = P[u, ε λ0 // CC[s λ0, u]]**

$$\begin{aligned} & \text{CWS} \left[ \epsilon \text{CW}[u], -\epsilon \text{CW}[uw] + s \epsilon \text{CW}[uw] + \frac{1}{2} \epsilon^2 \text{CW}[uw], \right. \\ & \quad s \epsilon \text{CW}[uuw] - \frac{1}{2} s^2 \epsilon \text{CW}[uuw] + \frac{1}{2} \epsilon^2 \text{CW}[uuw] - \frac{1}{2} s \epsilon^2 \text{CW}[uuw] - \frac{1}{6} \epsilon^3 \text{CW}[uuw] - \\ & \quad \left. s \epsilon \text{CW}[uww] + \frac{1}{2} s^2 \epsilon \text{CW}[uww] - \frac{1}{2} \epsilon^2 \text{CW}[uww] + \frac{1}{2} s \epsilon^2 \text{CW}[uww] + \frac{1}{6} \epsilon^3 \text{CW}[uww] \right] \end{aligned}$$

**t2 + t3**

$$\begin{aligned} & \text{CWS} \left[ s \text{CW}[u] + \epsilon \text{CW}[u], -s \text{CW}[uw] + \frac{1}{2} s^2 \text{CW}[uw] - \epsilon \text{CW}[uw] + s \epsilon \text{CW}[uw] + \frac{1}{2} \epsilon^2 \text{CW}[uw], \right. \\ & \quad \frac{1}{2} s^2 \text{CW}[uuw] - \frac{1}{6} s^3 \text{CW}[uuw] + s \epsilon \text{CW}[uuw] - \frac{1}{2} s^2 \epsilon \text{CW}[uuw] + \frac{1}{2} \epsilon^2 \text{CW}[uuw] - \\ & \quad \frac{1}{2} s \epsilon^2 \text{CW}[uuw] - \frac{1}{6} \epsilon^3 \text{CW}[uuw] - \frac{1}{2} s^2 \text{CW}[uww] + \frac{1}{6} s^3 \text{CW}[uww] - s \epsilon \text{CW}[uww] + \\ & \quad \left. \frac{1}{2} s^2 \epsilon \text{CW}[uww] - \frac{1}{2} \epsilon^2 \text{CW}[uww] + \frac{1}{2} s \epsilon^2 \text{CW}[uww] + \frac{1}{6} \epsilon^3 \text{CW}[uww] \right] \end{aligned}$$

**t1 ≡ t2 + t3**

True