```
SetDirectory["C:/drorbn/AcademicPensieve/2013-02/Sheffield/"]
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C:\drorbn\AcademicPensieve\2013-02\Sheffield

StandardAlexander

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & X-1 & 0 & -X \\ -1 & X & 0 & 0 & 0 & 0 & 1-X & 0 \\ 0 & -1 & X & 0 & 1-X & 0 & 0 & 0 \\ X-1 & 0 & -X & 1 & 0 & 0 & 0 & 0 \\ 0 & 1-X & 0 & -1 & X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -X & 1 & 0 & X-1 \\ 0 & 0 & 1-X & 0 & 0 & -1 & X & 0 \\ 0 & 0 & 0 & X-1 & 0 & 0 & -X & 1 \end{pmatrix}$$

```
[[1 ;; 7, 1 ;; 7]] // Det
```

StandardAlexander

$$-1 + 4 X - 8 X^2 + 11 X^3 - 8 X^4 + 4 X^5 - X^6$$

Initialization

```
βSimp = Factor; SetAttributes[βCollect, Listable];
βCollect[B[ω_, Λ_]] := B[βSimp[ω],
   Collect[Λ, h_, Collect[#, t_, βSimp] &]];
βForm[B[ω_, Λ_]] := Module[{ts, hs, M},
   ts = Union[Cases[B[ω, Λ], t_{u_} :→ u, Infinity]];
   hs = Union[Cases[B[ω, Λ], h_{x_} :→ x, Infinity]];
   M = Outer[βSimp[Coefficient[Λ, h_{#1} t_{#2}]] &, hs, ts];
   PrependTo[M, t_# & /@ ts];
   M = Prepend[Transpose[M], Prepend[h_# & /@ hs, ω]];
   MatrixForm[M]];
βForm[else_] := else /. β_B :→ βForm[β];
Format[β_B, StandardForm] := βForm[β];
```

Program

```
⟨μ_⟩ := μ /. t_ → 1;
tm_{u_ v_→w_}[β_] := βCollect[β /. t_{u|v} → t_w];
hm_{x_ y_→z_}[B[ω_, Λ_]] := Module[
   {α = D[Λ, h_x], β = D[Λ, h_y], γ = Λ /. h_{x|y} → 0},
   B[ω, (α + (1 + ⟨α⟩) β) h_z + γ] // βCollect];
sw_{u_ x_}[B[ω_, Λ_]] := Module[{α, β, γ, δ, ϵ},
   α = Coefficient[Λ, h_x t_u]; β = D[Λ, t_u] /. h_x → 0;
   γ = D[Λ, h_x] /. t_u → 0;    δ = Λ /. h_x | t_u → 0;
   ϵ = 1 + α;
   B[ω * ϵ, α (1 + ⟨γ⟩ / ϵ) h_x t_u + β (1 + ⟨γ⟩ / ϵ) t_u
         + γ / ϵ h_x              + δ - γ * β / ϵ
    ] // βCollect];
gm_{a_ b_→c_}[β_] := β // sw_{ab} // hm_{ab→c} // tm_{ab→c};
B /: B[ω1_, Λ1_] B[ω2_, Λ2_] := B[ω1 * ω2, Λ1 + Λ2];
Rp_{a_ b_} := B[1, (X - 1) t_a h_b];
Rm_{a_ b_} := B[1, (X^-1 - 1) t_a h_b];
```

htt

$$\left\{\beta = \mathbf{B}\left[\omega,\ \mathbf{Sum}\left[\alpha_{10\,i+j}\,\mathbf{t_i}\,\mathbf{h_j},\ \{\mathbf{i}, \{1, 2, 3\}\},\ \{\mathbf{j}, \{4, 5\}\}\right]\right],\ (\beta\ //\ \mathbf{tm}_{12\to 1}\ //\ \mathbf{sw}_{14}) == (\beta\ //\ \mathbf{sw}_{24}\ //\ \mathbf{sw}_{14}\ //\ \mathbf{tm}_{12\to 1})\right\}$$

htt

$$\left\{\begin{pmatrix} \omega & h_4 & h_5 \\ t_1 & \alpha_{14} & \alpha_{15} \\ t_2 & \alpha_{24} & \alpha_{25} \\ t_3 & \alpha_{34} & \alpha_{35} \end{pmatrix}, \text{True}\right\}$$

R3

$$\{\mathbf{Rm}_{51}\,\mathbf{Rm}_{62}\,\mathbf{Rp}_{34}\ //\ \mathbf{gm}_{14\to 1}\ //\ \mathbf{gm}_{25\to 2}\ //\ \mathbf{gm}_{36\to 3},\ \mathbf{Rp}_{61}\,\mathbf{Rm}_{24}\,\mathbf{Rm}_{35}\ //\ \mathbf{gm}_{14\to 1}\ //\ \mathbf{gm}_{25\to 2}\ //\ \mathbf{gm}_{36\to 3}\}$$

R3

$$\left\{\begin{pmatrix} 1 & h_1 & h_2 \\ t_2 & -\frac{-1+X}{X} & 0 \\ t_3 & \frac{-1+X}{X} & -\frac{-1+X}{X} \end{pmatrix}, \begin{pmatrix} 1 & h_1 & h_2 \\ t_2 & -\frac{-1+X}{X} & 0 \\ t_3 & \frac{-1+X}{X} & -\frac{-1+X}{X} \end{pmatrix}\right\}$$

8_17-1

$$\beta = \mathbf{Rm}_{12,1}\,\mathbf{Rm}_{27}\,\mathbf{Rm}_{83}\,\mathbf{Rm}_{4,11}\,\mathbf{Rp}_{16,5}\,\mathbf{Rp}_{6,13}\,\mathbf{Rp}_{14,9}\,\mathbf{Rp}_{10,15}$$

8_17-1

$$\begin{pmatrix} 1 & h_1 & h_3 & h_5 & h_7 & h_9 & h_{11} & h_{13} & h_{15} \\ t_2 & 0 & 0 & 0 & -\frac{-1+X}{X} & 0 & 0 & 0 & 0 \\ t_4 & 0 & 0 & 0 & 0 & 0 & -\frac{-1+X}{X} & 0 & 0 \\ t_6 & 0 & 0 & 0 & 0 & 0 & 0 & -1+X & 0 \\ t_8 & 0 & -\frac{-1+X}{X} & 0 & 0 & 0 & 0 & 0 & 0 \\ t_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1+X \\ t_{12} & -\frac{-1+X}{X} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t_{14} & 0 & 0 & 0 & 0 & -1+X & 0 & 0 & 0 \\ t_{16} & 0 & 0 & -1+X & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

8_17-2

$$\mathbf{Do}[\beta = \beta\ //\ \mathbf{gm}_{1k\to 1},\ \{\mathbf{k}, 2, 10\}];\ \beta$$

8_17-2

$$\begin{pmatrix} \frac{1}{X} & h_1 & h_{11} & h_{13} & h_{15} \\ t_1 & -\frac{(-1+X)\,(1+X)}{X} & -(-1+X)\,\left(1-X+X^2\right) & (-1+X)\,\left(1-X+X^2\right) & -1+X \\ t_{12} & -\frac{-1+X}{X} & 0 & 0 & 0 \\ t_{14} & -1+X & \frac{(-1+X)^2\,\left(1-X+X^2\right)}{X} & -\frac{(-1+X)^2\,\left(1-X+X^2\right)}{X} & 0 \\ t_{16} & \frac{-1+X}{X} & (-1+X)^2 & -\frac{(-1+X)^3}{X} & 0 \end{pmatrix}$$

8_17-3

$$\mathbf{Do}[\beta = \beta\ //\ \mathbf{gm}_{1k\to 1},\ \{\mathbf{k}, 11, 16\}];\ \beta$$

8_17-3

$$\left(-\frac{1-4X+8X^2-11X^3+8X^4-4X^5+X^6}{X^3}\right)$$