Vladimir Dotsenko on "Shuffle Operads"

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6:42 AM

Operads today have only positive arities — no constants

Fin: the category of non-empty finite sets.
Ord: Linearly ordered finite sets w/ order pres. bijections.

Symmetric Collections:
Functors $\mathrm{Fin}^{op} \to \mathrm{Vect}$

Non-Symmetric collections:
Functors $\mathrm{Ord}^{op} \to \mathrm{Vect}$

In either case such a functor is determined by its value $P(n) := P(\{1,\dots,n\})$

Perhaps I should rephrase meta-groups in a similar language ?

Monoidal structures

Symmetric:

$$(P\circ Q)(I) = \bigoplus_{k\geq 1} P(\underline{k}) \otimes_{S_k} \bigoplus_{f: I \twoheadrightarrow \underline{k}} Q(f^{-1}(1)) \otimes \dots \otimes Q(f^{-1}(k))$$

Non-symmetric:

$$(P\circ Q)(I) = \bigoplus_{k\geq 1} P(\underline{k}) \otimes \bigoplus_{\substack{f: I \twoheadrightarrow \underline{k} \\ \text{non-decreasing}}} (\text{ same })$$

An operad is a monoid in one of these categories.

discussion skipped

$\mathrm{Lie} = \mathrm{Free}\left(\overset{1\ 2}{Y} = - \overset{2\ 1}{Y} \right) / \mathrm{Jacobi}$

Much skipped. on the def of "shuffle operads", which allow for a partial non-symmetric description of symmetric operads.

In the Lie case recover Lyndon words.

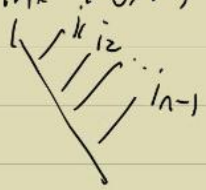
Subject More Dotsenko notes.

$\mathrm{Free}_{sh}\left(\overset{1\ 2}{Y}\right) / \left(\overset{1\ 2\ 3}{Y}\right)$

— close to Lyndon.

$\mathrm{Free}_{sh}\left(\overset{1\ 2}{Y}\right) / \left(\overset{1\ 2\ 3}{Y}\right)$

— get "shuffled trees" of the form



Example 2 Associative algebras as symmetric operads....