Vallette: The Three Graces

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11:36 AM

No Video!

symmetric AS

$$\mathcal{L}ie \xrightarrow{\ \overline{}\ } Ass \longrightarrow Com$$

$$\mathrm{Y}^{12} \mapsto \mathrm{Y}^{12} - \mathrm{Y}^{21} \qquad \mathrm{Y}^{12}, \mathrm{Y}^{21} \longmapsto \mathrm{Y}$$

these are morphisms of operads that respect quadratic data; hence

$$\mathcal{L}ie^{¡} \rightarrowtail Ass^{¡} \twoheadrightarrow Com^{¡}$$

hence

$$\Omega \mathcal{L}ie^{¡} \rightarrowtail \Omega Ass^{¡} \twoheadrightarrow \Omega Com^{¡}$$

hence

$$L_\infty\text{-alg} \twoheadleftarrow A_\infty\text{-alg} \hookleftarrow C_\infty\text{-alg}$$

Now the details ...

$$L_\infty\text{-alg} = \Omega \mathcal{L}ie^{¡}\text{-alg}$$

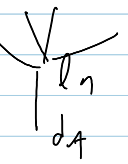
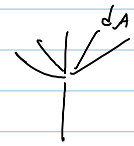
$$\| $$

$$S^{-1} \otimes Com^{*} = (0, \mathrm{Y}, \overset{\text{alt.}}{\mathrm{Y}}, \overset{\text{alt}}{\mathrm{Y}})$$

deg 0 1 2

So $\mathrm{Y}^{1 \cdots n} : A^{\otimes n} \xrightarrow{l_n} A$, skew-symmetric deg $n-2$

$$d_2\, \mathrm{Y} = \sum_{\substack{p+q=n+1 \\ |I|=p,\ |J|=q-1 \\ I \cup J = n}} \pm\, \mathrm{Y}^{\overbrace{}^{I}}_{\ \ \ \mathrm{Y}^{\overbrace{}^{J}}}$$

$\partial(l_n) =$  $+ \sum$ 

These two need to be compatible, meaning

$$\partial(l_n) = \sum_{I \cup J = \underline{n}} \operatorname{sign}(\sigma)(-1)^{(p-1)q}\, l_q \circ_1 l_p$$

w/ σ the shuffle defined by I, J.

<u>Comment</u> If $\{\mu_n\}$ form an A_∞-algebra, then $\{l_n := \sum_{\sigma \in S_n} (-1)^{\sigma} \mu_n^{\sigma}\}$ form an L_∞-algebra.

C_∞-algebras have operations labeled by elements of the free Lie algebra. They can also be described as A_∞-algs which satisfy a relation having to do w/ shuffles.

E_∞: relax both commutativity & associativity
