

More on the previous lecture ----

$$\begin{array}{ccccc} \text{Hom}_{\text{op}}(\mathcal{T}(S^{-1}\mathcal{E}), \mathcal{P}) & \cong & \text{Hom}_{\mathcal{E}}^{-1}(\mathcal{E}, \mathcal{P}) & \cong & \text{Hom}_{\text{coop}}(\mathcal{E}, \mathcal{T}^c(S\bar{\mathcal{P}})) \\ \downarrow & & \downarrow & & \downarrow \\ \text{Hom}_{\text{dgop}}(\mathcal{L}\mathcal{E}, \mathcal{P}) & \cong & \text{Tw}(\mathcal{E}, \mathcal{P}) & \cong & \text{Hom}_{\text{dgcoop}}(\mathcal{E}, B\mathcal{P}) \\ \downarrow & & \downarrow & & \downarrow \\ \text{QI}(\mathcal{L}\mathcal{E}, \mathcal{P}) & \cong & \text{Kos}(\mathcal{E}, \mathcal{P}) & \cong & \text{QI}(\mathcal{E}, B\mathcal{P}) \end{array}$$

⋮

Things simplify if the operads are "quadratic".

See video, even if this phrase appears here too often.

Yet I'm a low-faith fellow, and there's only that much I can go without knowing that what's being done must be done for the purpose of some good cause. In other words, I cannot work unmotivated.

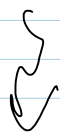
And, perhaps embarrassingly, I still don't understand the motivation for "resolutions", even in the most basic sense.

I guess, even before "what's a resolution", comes "what's a module". I don't even know the answer to that.

Def. Gerstenhaber algebra:

$$\ast : A^{\otimes 2} \rightarrow A \quad \text{deg } 0 \text{ commutative product.}$$

$$[\ , \ ]: (sA)^{\otimes 2} \rightarrow sA \quad \text{deg } 0 \text{ bracket.} \\ \text{(skew symmetric)}$$



$$\langle \ , \ \rangle: A^{\otimes 2} \rightarrow A \quad \text{deg } 1 \text{ symmetric} \dots$$

... The homology of the little disk operad.

Does the little disk operad ever say something truly useful about honest braids?

"The Chevalley-Eilenberg complex of a free Lie algebra is acyclic"

Rewriting methods

Associative algebras

I should try to understand this as "Gaussian elimination in the bulk".

$$A(V, R) = S(V) = T(V) / \langle x \otimes y - y \otimes x \rangle$$

1. Choose a totally ordered basis for  $V$

$$V = \{x_1 < x_2 < x_3 < \dots\}$$

2. Consider the <sup>graded</sup> lexicographic order, or any other order w/ increases under concatenation

$$x_1 < x_2 < x_3$$

$$x_1 x_1 < x_1 x_2 < \dots$$

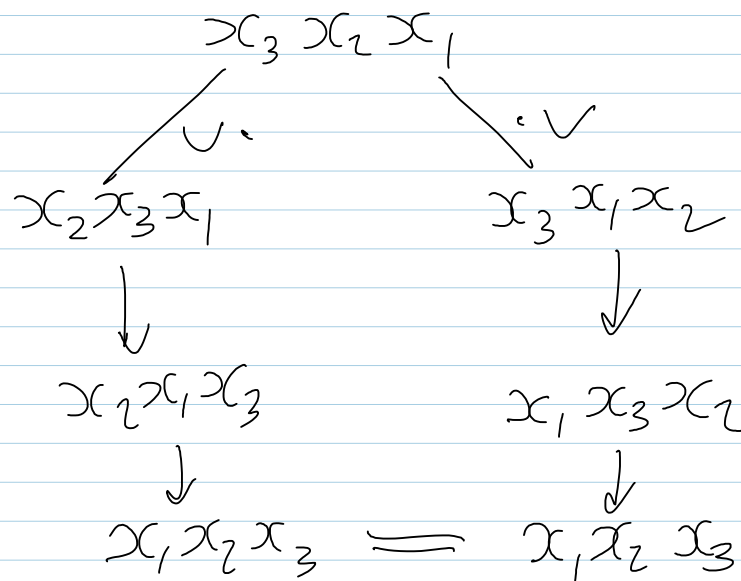
Choose a basis for  $R$  s.t.

the leading term [highest] in every relation does not appear in any other relation.

3. Interpret the relations as decreasing rewriting rules.

The "critical monomials" are those in which  $XYZ$ , both  $XY$  &  $YZ$  are reducible.

4. Write the diamond for each critical monomial



"Confluence" — going both ways we got to the same place.

Thm IF such a procedure exists, which always leads to confluences, then the algebra is Koszul.

A basis  $A$  is "words w/ no leading terms"