Vallette: Koszul Duality for Operads, 1

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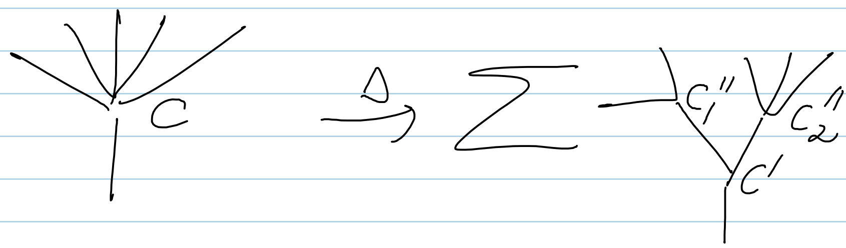
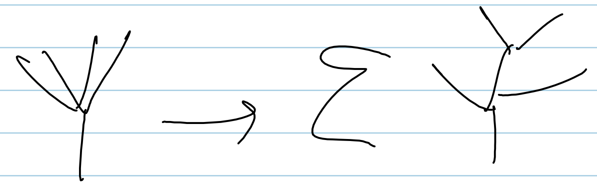
1. Bar and cobar constructions for operads.

$\mathcal{P}$: operad $\quad \mathcal{P}\circ\mathcal{P}\xrightarrow{\gamma}\mathcal{P}$

$\mathcal{C}$: co-operad $\quad \mathcal{C}\xrightarrow{\Delta}\mathcal{C}\circ\mathcal{C}$



$\Delta_{(1)}:\mathcal{C}\to\mathcal{C}\overset{\circ}{\otimes}\mathcal{C}$ the "infinitesimal coproduct":



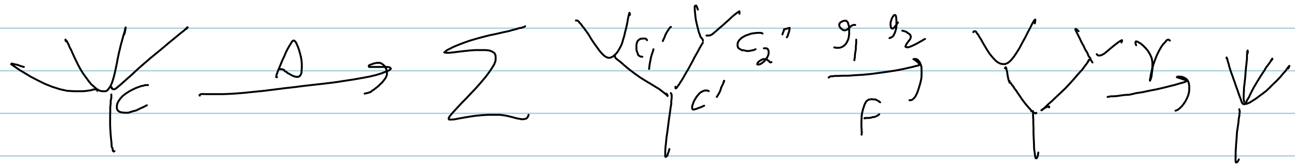
just one split

Def $\mathrm{Hom}(\mathcal{C},\mathcal{P})=\{\mathrm{Hom}(\mathcal{C}(n),\mathcal{P}(n))\}_n$

has the structure of an operad:

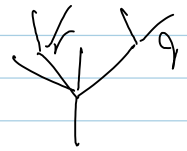
Given $f, g_1 \dots g_n \in \mathrm{Hom}(\mathcal{C},\mathcal{P})$ do

$\gamma(f, g_1 \dots g_n) =$



Recall $p \star q = \sum p \circ_i q$ in an operad

— not quite associative:

$(p*q)*r - p*(q*r) = \sum$ 

So the "associator" is symmetric in q & r

A "pre-Lie algebra"

Define $p*q \pm q*p =: [p,q]$ — it is a Lie bracket!

So operad $\Rightarrow$ pre-Lie alg $\Rightarrow$ Lie alg.

In the symmetric case convolutions don't an operad make, yet $*$ & $[,]$ remain defined. So MC makes sense, given a dgop & dgcop

We now try to represent twisting morphisms...

see video

... get a cobar construction for operads.

... and a bar construction likewise.

much more in video.

Motivation. Look for a quasi-free resolution of an operad P .... more in video.

....

Eventually get A_∞ algebras, in video.