Proving the J-property

January-22-13

12:04 PM

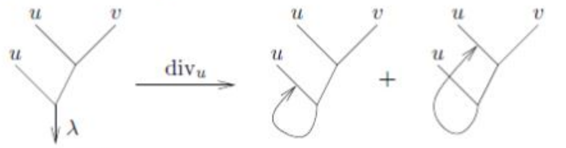
From the paper: $CC_{u,\bar{u}}^{\mathrm{bch}(\lambda_x,\lambda_y)} = CC_{u,\bar{u}}^{\lambda_x} \,/\!\!/\, CC_{\bar{u},\bar{u}}^{\lambda'_y}$

The Meta-Cocycle J. Set $J_u(\lambda) := J(1)$ where

$$J(0) = 0, \qquad \lambda_s = \lambda \,/\!\!/\, CC_u^{s\lambda},$$

$$\frac{dJ(s)}{ds} = (J(s) \,/\!\!/\, \mathrm{der}(u \mapsto [\lambda_s, u])) + \mathrm{div}_u \lambda_s,$$

and where $\mathrm{div}_u \lambda := \mathrm{tr}(u\sigma_u(\lambda))$, $\sigma_u(v) := \delta_{uv}$, $\sigma_u([\lambda_1,\lambda_2]) := \iota(\lambda_1)\sigma_u(\lambda_2) - \iota(\lambda_2)\sigma_u(\lambda_1)$ and ι is the inclusion $FL \hookrightarrow FA$:



Claim. $CC_u^{\mathrm{bch}(\lambda_1,\lambda_2)} = CC_u^{\lambda_1} \,/\!\!/\, CC_u^{\lambda_2 /\!\!/ CC_u^{\lambda_1}}$ and

$$J_u(\mathrm{bch}(\lambda_1,\lambda_2)) = J_u(\lambda_1) \,/\!\!/\, CC_u^{\lambda_2 /\!\!/ CC_u^{\lambda_1}} + J_u(\lambda_2 \,/\!\!/\, CC_u^{\lambda_1}),$$

and hence tm, hm, and tha form a meta-group-action.

with $(\lambda, w) /\!\!/ tha^{ux} :=$

$(\lambda, w) /\!\!/ CC_u^{\lambda_x} + (0, J_u(\lambda_x))$

Also with $\frac{d}{ds} J_u(s\lambda)\big|_{s=0} = \mathrm{div}_u \lambda$

and

$\frac{d}{ds} J_u(\lambda /\!\!/ v \mapsto sv) = ?$

$$\mu /\!\!/ hm_z^{xy} /\!\!/ tha^{uz} \stackrel{?}{=} \mu /\!\!/ tha^{ux} /\!\!/ tha^{uy} /\!\!/ hm_z^{xy}$$

$$J_u(\mathrm{bch}(\lambda_x, \lambda_y)) \stackrel{?}{=} J_u(\lambda_x) /\!\!/ CC_u^{\lambda_y /\!\!/ CC_u^{\lambda_x}} + J_u(\lambda_y /\!\!/ CC_u^{\lambda_x})$$

The t-action equation yields

$$J_w(\lambda / u, v \to w)$$

$$= \left[J_u(\lambda) /\!\!/ CC_v^{\lambda /\!\!/ CC_u^{\lambda}} + J_v(\lambda /\!\!/ CC_u^{\lambda}) \right] /\!\!/ u, v \to w$$

Aside — what if we used the "prior splice":

$$(\lambda, w) /\!\!/ tha^{ux} := (\lambda, w + P_u(\lambda_x)) /\!\!/ CC_u^{\lambda_x} \;?$$

The h-action equation becomes:

$$P_u(\mathrm{bch}(\lambda_x, \lambda_y)) /\!\!/ CC_u^{\mathrm{bch}(\lambda_x, \lambda_y)}$$

$$= P_u(\lambda_x) /\!\!/ CC_u^{\lambda_x} /\!\!/ CC_u^{\lambda_y /\!\!/ CC_u^{\lambda_x}} + P_u(\lambda_y /\!\!/ CC_u^{\lambda_x}) /\!\!/ CC_u^{\lambda_y /\!\!/ CC_u^{\lambda_x}}$$

Cancelling the $CC_u^{\lambda_y /\!\!/ CC_u^{\lambda_x}}$ everywhere gives

$$P_u(\mathrm{bch}(\lambda_x, \lambda_y)) /\!\!/ CC_u^{\lambda_x} = P_u(\lambda_x) /\!\!/ CC_u^{\lambda_x} + P_u(\lambda_y /\!\!/ CC_u^{\lambda_x})$$