Pierre Lochak on "Grothendieck-Teichmuller Theory"

January-08-13
9:00 AM

First inputs in G.T. theory

Grothendieck 1983
Ihara Annals 1986
Drinfel'd 1989
Deligne $P^1 \setminus 3$pts, MSRI 1980

Three messages:

* Large Galois reps are often faithful.
* 2-level principle
* the theory is or should be "non-linear"

Belyi (1976) Any smooth curve X over $\overline{\mathbb{Q}}$ can be realized as a covering of $\mathbb{P}^1$ ramified over at most $0, 1, \infty$.

[SGA1]: X geom. connected scheme over $\mathbb{K}$,

$\pi_1^{geom}(X) = \pi_1(X \otimes \overline{\mathbb{K}})$, then

$$1 \longrightarrow \pi_1^{geom}(X) \longrightarrow \pi_1(X) \longrightarrow \underset{\underset{G_{\mathbb{K}}}{\parallel}}{Gal(\mathbb{K})} \longrightarrow 1$$

is exact.

With $X = P^* := \mathbb{P}^1 \setminus \{0, 1, \infty\}$ get

$$G_{\mathbb{K}} \longrightarrow \mathrm{Out}(\pi_1^{geom}(X))$$

Cor $$G_{\mathbb{Q}} \longrightarrow \mathrm{Out}(\pi_1(P^*))$$
$$\pi_1(P^*) = \hat{F}_2 \leftarrow \text{profinite completion.}$$

Rest in video: How Gal acts on $\hat{\pi}_1(P^*)$.
Explicit constraints on the image of $G_{\mathbb{Q}}$ in $\mathrm{Out}^*(\hat{F}_2)$ — this is the GT group.