Merkulov on "Grothendieck-Teichmuller groups, graph complexes and deformation quantization"

January-15-13
8:07 AM

On Willwacher's theorem an its applications.

1. Lie algebra V also $V[1]$

$\underbrace{\mathrm{Lie}}_{\text{operad}} \xrightarrow{\rho} \mathrm{End}(V)$

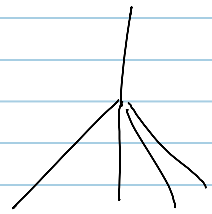
Any morphism of operad $\ell_1 \to \ell_2$ is a MC-element in some Lie_∞ algebra

$\mathrm{Def}(\mathrm{Lie} \xrightarrow{\circ} \mathrm{End}(V))$

[Read paper by Merkulov & B. Vallette]

$= \mathrm{Def}(\mathrm{Lie}_\infty \xrightarrow{\circ} \mathrm{End}(V)$

After shift:



$\mathrm{Def}(\mathrm{Lie}[1]$

$3-2n$

$\mathrm{Lie}_\infty\{1\}$

$\cong$

Fund. chains in some conf. space

See video

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2. Kontsevich graph complex.

M manifold, $C^\infty(M)$, $\mathrm{Der}(C^\infty(M))$, $[\,,\,]$

extends to

$$V := \left(\Lambda^*_{C^\infty(M)} T_M \, , [\, , \,] \right)$$

$\mathrm{Def}\left(\mathrm{Lie}[1] \longrightarrow \mathrm{End}_{\Lambda^* T_M} \right)$

more in video.

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what's Chevalley - Eilenberg ?

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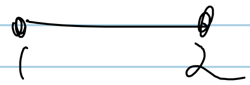
3. Gra , "the graph operad".

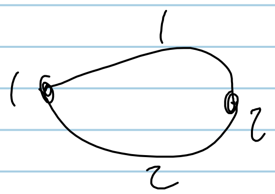
<u>Def 1</u> $G_{n,l}$: set of graphs Γ s.t.

1. $\# V(\Gamma) = n$, vertices are enumerated.
2. $\# E(\Gamma) = l$, and the set of edges is oriented: $or_\Gamma \in \Lambda^{top}(E(\Gamma))$

(loops allowed, multiple edges allowed)

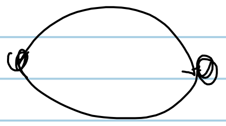
 $\in G_{2,1}$

 $\in G_{1,1}$

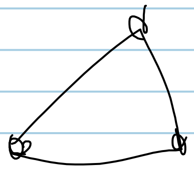
 $\in G_{2,2}$

$\mathrm{Gra} = \{ \mathrm{Gra}(n) \}$ a collection of graded S_n-modules

$\mathrm{Gra}(n) = \bigoplus_{l \geq 0} \mathbb{K} \langle G_{n,l} \rangle [l] \Big/$ orientation AS meaning that each edge has degree -1.

 $= 0$ as it has an odd automorphism.

$\Longrightarrow$ No multiple edges in $\mathrm{Gra}(n)$.

Example  degree $= -3$

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claim: Gra is an operad using

$(\Gamma_1, \Gamma_2) \xrightarrow{\circ_i}$ sum of ways of putting Γ_2 as the i'th vertex of Γ_1, lifting the edges of Γ_1 in all possible ways.

Gra has a representation in poly-vector fields on $\mathbb{R}^d$. see video ???

This is all "low algebra".

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no video from here.

$\mathrm{Def}(\mathrm{Lie}_\infty[1] \xrightarrow{\circ} P) = \bigoplus_{n \geq 1} P_{S_n}(n)$ invariants

$[a, b] = \sum a \circ_i b \pm \sum b \circ_j a$

$\mathrm{Def}(\mathrm{Lie}_\infty[1] \xrightarrow{\circ} \mathrm{Gra})$

Thm (Willwacher) $H^0(GC_2) = grt$