

Title. Polynomial Invariants of Groups.

Abstract. A "polynomial invariant" (also, a "finite type invariant") on a group G is a function on G with values in \mathbb{Z} (or actually, in any Abelian group A), whose linear extension to the group ring $\mathbb{Z}G$ of G vanishes on n -fold iterated differences of elements of G - namely on elements of the power I^n of the augmentation ideal $I = \langle g-h : g, h \in G \rangle$ of $\mathbb{Z}G$. This, of course, is an imitation and a generalization of the notion of a "polynomial" on a lattice or on \mathbb{R}^n ; these fellows too, can be defined combinatorially as "the things which vanish on iterated differences". There are lovely examples of groups whose theory of polynomial invariants is rich and interesting, and much still remains to be done.

Yet this is not what I want to talk about. The above is about polynomial invariants ON groups; I want to talk about polynomial invariants OF groups. For this, we first have to make the set of all groups (appropriate groups, at least) itself into a group (or maybe a meta-group).

When restricted to the fundamental groups of knot complements, the invariants I will talk about appear to be a complete evaluation of the BF quantum field theory on ribbon 2-knots in \mathbb{R}^4 , and they contain a "non-commutative" generalization of the classical Alexander polynomial which contains in it formulas for good old Alexander that are better than anything known before. Finally, everything's computable and I have the programs to prove it.