Ieke Moerdijk on "The Boardman-Vogt resolution and algebras up-to-homotopy"

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10:29 AM

Operads in $(\mathrm{Top}, \times)$ or in $(\mathrm{Sets}, \times)$

If $P$ is an operad a $P$-algebra is

$$(*) \qquad P(n) \times A^n \xrightarrow{\alpha_n} A \qquad \left[\text{satisfying some conditions}\right]$$

$Q \xrightarrow{\varphi} P$ operad morphism make every $P$-algebra into a $Q$-algebra.

For us operads are non-symmetric.

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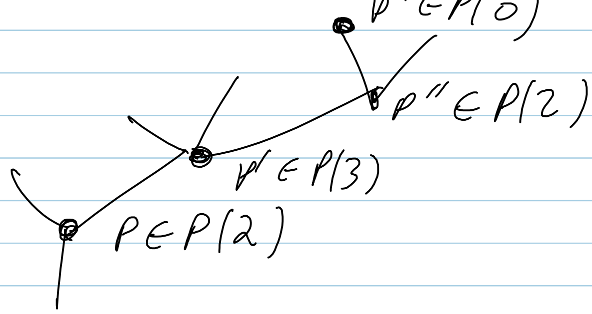
If we forget the equations in $(*)$, the resulting algebras are "controlled" by $\mathrm{Free}(P)$, whose definition is obvious, using finite, rooted, planar trees:




$\in \mathrm{Free}(P)(4)$

operad structure by "grafting".

There is also $\mathrm{Free}_I(P)$ which is

$$\mathrm{Free}(P) \Big/ \ \bullet 1 \in P_1 = |$$

There are maps of operads

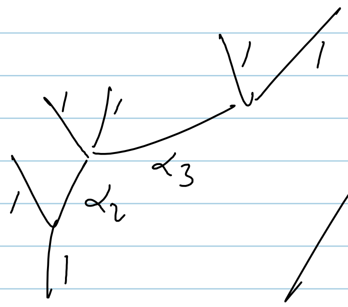
$$\begin{tikzcd}
\mathrm{Free}(P) \arrow[r] \arrow[dr, "\epsilon"'] & \mathrm{Free}_I(P) \arrow[r] \arrow[d, "\epsilon"] & W_I(P) \arrow[dl, "\epsilon"] \\
 & P &
\end{tikzcd}$$

So every $P$-algebra is also a $\mathrm{Free}(P)$-algebra

→ "Controls the equation up to homotopy":

$W_I(P)$ = "The Boardman-Vogt resolution"

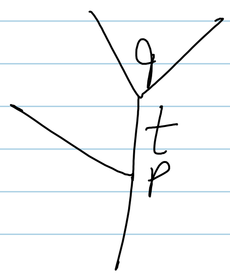
Elements of $W_I(P)$ are like elements of $\mathrm{Free}(P)$, with internal edges labeled with lengths $\in [0,1]$:




* edges of length 0 can be contracted.

* $\alpha_1 \,|\, \circ_I \,|\, \alpha_2 = |\, \max(\alpha_1, \alpha_2)$

(all non-internal edges are labeled 1)

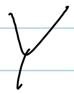


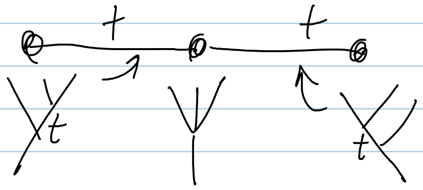

a homotopy between...

a $W_I(P)$ algebra is like a $P$-algebra, except eqn's only hold up to homotopy.

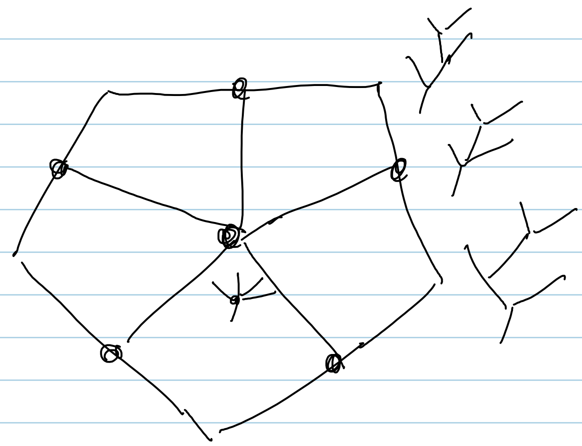
<u>Example</u> $P = ASS$ (assoc. algebra w/no unit)

$W(Ass)(1) = pt.$

$W(Ass(2)) =$ 

$W(Ass(3)) =$ 


a "subdivided homotopy"

$W(Ass(4)) =$ 

More in video.