Hiroaki Nakamura on "Arithmetic and Topological Problems in Universal Monodromy Representations of Galois-Teichmuller (Modular) Groups"

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6:35 AM

1. Topological motivation.
2. Some known / unknown results.
3. Remarks.

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1. Dehn-Nielson isom:

$$\rho_{DN} : M_{g,n} \xrightarrow{\sim} Out^+(\Pi_{g,n})$$

mapping class group

$$\Pi_{g,n} = \Big\langle \underbrace{x_1 \ldots x_{2g}}_{\text{weight } 1} ; \prod_{i=1}^{g} [x_i, x_{g+i}] , \underbrace{z_1 \ldots z_n}_{\text{weight } 2} \cdot \prod z_j = 1 \Big\rangle$$

ρ_{DN} maps a "hard" group into the out of an easy group.

Graded Lie version using the weight filtration

$$\Pi = \Pi_{g,n} \supset \Pi(1) \supset \Pi(2) \supset \ldots$$

with $\Pi(1) = [\Pi, \Pi] \langle z_1 \ldots z_n \rangle$

$\Pi(2) = [\Pi, \Pi(1)]$ etc.

likewise

$$M_{g,n} = M_{g,n}(0) \supset M_{g,n}(1) \supset \ldots$$

where $\qquad \forall x \in \Pi_{g,n}(1)$

$$\mathcal{M}_{g,n}(k) = \left\{ \sigma \in \mathrm{Aut}^+(\pi_{g,n}) : \frac{\sigma(x)\cdot x^{-1} \in}{\pi_{g,n}(k+1)} \right\}$$

$\mathcal{M}_{g,n}(1) :=$ Torelli subgroup

$$\mathcal{M}_{g,n} / \mathcal{M}_{g,n}(1) = Sp(2g) \times S_n$$

$Sp(2g)$ acts on $\bigoplus \mathbb{Z} x_i$; S_n permutes points

$$\text{Lie}\,\rho_{DN} : \bigoplus_{k=1}^{\infty} g^k \mathcal{M}_{g,n} \hookrightarrow \bigoplus_{k=1}^{\infty} \mathrm{Sder}^k(\mathrm{Gr}\,\pi_{g,n})$$

not an isomorphism

Hain (1998) – LHS is "almost quadratic"

- - - - continued on video. . . .