Andrew Stacey on "Calculus of Functors and Knot Theory"

January-21-13
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* Extremely brief review of "calculus of functors"
* Review of Embedding calculus.
* Connection to knots links etc: The good, the bad, the ugly.
  — Calculus of functor ends up playing a relatively minor role.

Based on work of Ismar Volić & Brian Munson.

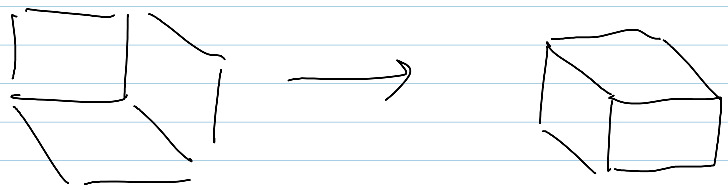
Calculus of functors:

$F : \mathcal{C} \longrightarrow Top$ (w/ nice properties)

construct "approximations" $T_k F : \mathcal{C} \to Top$ w/ nat. trans.

$$\begin{tikzcd} & \vdots \arrow[d, dashed] \\ & T_2 F \arrow[d, dashed] \\ F \arrow[ur] \arrow[r] \arrow[dr] & T_1 F \arrow[d, dashed] \\ & T_0 F \end{tikzcd}$$

$T_k F$ "sees F up to k-cubes"



strongly homotopy co-Cartesian — homotopy Cartesian

Ideal: 1. holim $T_k F$ should converge in the sense that homotopy groups should stabilize.
2. holim $T_k F$ should be weakly equiv. to F.

Manifold calculus — Embeddings.

Source mfld M^m, target mfld N^n

use $\mathcal{O}(M)^{op}$: open subsets of M, morphisms are "reverse inclusions"

F is $\mathrm{Emb}(-, N) : \mathcal{O}(M)^{op} \to$ spaces.

Nice properties:

1. $U \xrightarrow[\supseteq]{} V$ isotopy $\Rightarrow$ homotopy equiv. $\mathrm{Emb}(U, N) \xrightarrow{\sim} \mathrm{Emb}(V, N)$

2. If $U = \bigcup U_i$ with $U_i \subset U_{i+1}$, then

$$\mathrm{Emb}(U, N) = \operatorname{holim} \mathrm{Emb}(U_i, N)$$

Convergence result: The functor $U \mapsto \mathrm{Emb}(U, N)$ is $(n-2)$-analytic with excess $3-n$

$$\mathrm{Emb}(M, N) \to T_k \mathrm{Emb}(M, N)$$

is $(k(n-m-2)+1-m)$-connected. So if $n-m-2>0$ then the above is a weak equivalence.

Applications to knots &c.

Knot space: $\mathrm{Emb}(S^1, N^3)$

Link space: $\mathrm{Emb}(\coprod_\ell S^1, N^3)$

Formulas give no information about convergence!

We want to study $\pi_0(\text{knot space})$ by studying

$$H^0(\text{knot space}, A)$$

We have $H^0(T_k \text{ knot space}) \longrightarrow H^0(\text{knot space})$.

How good a source of invariants is this?

$H^0(T_k \text{ knot space})$ "are" F.T. invariants.

<u>Step 1</u> Build a model for $T_k \text{ knot space}$ — simplicial, pieces are configuration spaces of points and directions in $\mathbb{R}^3$.

<u>Step 2</u> Compactify.

<u>Step 3</u> Apply big machinary (spectral sequences)

<u>Step 4</u> Analyze the SS & show convergence results, identify the limits.